Studies of the geocenter motion using 16-years DORIS data

S.P. Kuzin a, S.K. Tatevian a,*, S.G. Valeev b, V.A. Fashutdinova b

a Institute of Astronomy RAS, 48 Pyatnitskaya str., 119017, Moscow, Russian Federation
b Ulyanovsk State Technical University, 32 Severny Venetz str., 432027 Ulyanovsk, Russian Federation

Received 6 October 2009; received in revised form 18 June 2010; accepted 22 June 2010

Abstract

An accuracy of geocenter motion estimation is strongly dependent on the geodetic network size and stations distribution over the Earth’s surface. From this point of view DORIS system has an advantage, as its ground network of beacons consists of more than 50 sites, equally distributed over the Earth’s surface. Aiming to study variations of the geocenter movements, the results of DORIS data analysis for the time span 1993.0–2009.0 (inawd06.snx series), performed at the Analysis Centre of the Institute of astronomy of the Russian Academy of Sciences, have been used. DORIS data processing was made with GIPSY/OASIS II software, developed by Jet Propulsion Laboratory and modified for DORIS data processing by Institute Geographique National. Standard deviations of stations coordinates are estimated at the level 0.5–4.0 cm (internal consistency), depending on the number of satellites used in the solution. RMS of estimated components of the DORIS satellites orbits, compared with the solutions of other IDS analysis centres, do not exceed 1–2 cm. Weekly solutions for coordinates have been transformed from free network solutions (inawd06.snx series) to a well defined terrestrial reference frame ITRF2005 with the use of seven parameters of Helmert transformation, which were examined with a view to study variations of the geocenter movements (ina05wd01.geoc time series). In order to estimate linear trend, amplitudes, periods and phases of geocenter variation a method of linear regression was applied. The evaluated amplitudes of annual variations are of the order of 5–7 mm for $X$ and $Y$ components and 27–29 mm for $Z$ component. Semi-annual amplitudes are also noticeable in all components (1–3 mm for $X$, $Y$ and $Z$ components). Secular trends in the DORIS geocenter coordinates are: $-1.2$, $-0.1$ and $-0.3$ mm/year for $X$, $Y$ and $Z$ directions respectively.

Spectral analysis of the same set of DORIS data (1993.0–2009.0) with the method of dynamic regression modeling allowed to find out several other harmonics with periods of 2, 3, 5 years and valuable amplitudes (1–3 mm for $X$, $Y$ components and about 10 mm for $Z$ component) and to develop a model of the geocenter dynamics. The simulated values of geocenter coordinates, estimated for the 2008 year with the use of shortcut model, limited by 1993–2007 data, coincide with “observable” (calculated with use of DORIS measurements) data with the correlation coefficient 0.7–0.8.

© 2010 COSPAR. Published by Elsevier Ltd. All rights reserved.

Keywords: Geocenter; DORIS measurements; Spectral analyses; Mathematical model

1. Introduction

Conventionally, the origin of the ITRF is defined to be at the center of mass (CM) of the entire Earth system, including the solid earth, oceans, atmosphere and continental waters (McCarthy and Petit, 2004). In reality the global geodetic network is fixed to the solid Earth crust and its origin coincides with the center of the solid earth surface figure (CF). Space geodetic techniques have demonstrated the 3-D vector displacement of the center of the figure (CF) relative to the center of mass (CM) at the level of a few millimeters to centimeters for the periods from diurnal to seasonal (Crétaux et al., 2002; Tatevian et al., 2004; Gobinddass et al., 2009a). These variations are defined as the geocenter motion and directly affect estimates of all space geodetic measurements that use the ITRF as a reference system.
As it was shown by several authors (Altamimi et al., 2006; Lavalle et al., 2006) an accuracy of geocenter vector estimation is strongly dependent on the geodetic network size and station distribution over the Earth’s surface. Almost 80% of sites of the International Global Navigation Satellite Service (GNSS-IGS) are located in the Northern hemisphere, and the result is that the largest differences of the geocenter position vector are in the estimated Z-components. The inequality in the X and Y directions varies only up to 15%, but there are still noticeable tendency toward sites being located in the Europe (X-axis) and in the North America (Y-axis). From this point of view the DORIS (Doppler Orbitography and Radio positioning Integrated by Satellites) system (Tavernier et al., 2006; Willis et al., 2006a,b) has an advantage, as its ground network of beacons consists of more than 50 sites, well distributed over the Earth’s surface. DORIS is a satellite system, developed to support high accuracy orbit determination for altimetry measurements of the sea level and ground beacon positioning. The space segment now is accounted for six satellites (SPOT 2, SPOT 4, SPOT 5, ENVISAT, JASON1 and JASON2).

The Analysis Center (INA) of the Institute of astronomy, RAS (INASAN) performs DORIS data analysis with the use of GIPSY/OASIS II software, developed by Jet Propulsion Laboratory (JPL) (Webb and Zumberge, 1997) and significantly expanded for DORIS applications (Willis et al., 2005) by joint IGN/JPL cooperation. The station coordinates, estimated on daily basis using all available satellites with DORIS equipment, are combined into weekly solutions, projected (removing of the indetermination due to a loosely definition of the terrestrial reference frame) and transformed to the well-defined coordinate system (typically ITRF) with the use of seven parameters of Helmert transformation (Kuzin and Tatevian, 2000, 2002). Three translations parameters are the components of the geocenter 3-D vector. This method of the geocenter estimation is known as “geometric” or network shift approach.

2. Computation standards

Taking into account recommendations of the International Earth Rotation and Reference Systems Service (IERS) and International DORIS service (IDS) (Tavernier et al., 2006), a reprocessing of the DORIS data for the period of 1993.0–2009.0 has been performed aiming to obtain a unified coordinated solution of the IDS analysis centers for the developing of the new version of the Terrestrial Reference Frame – ITRF2008. All previous calculations with DORIS data, made at the INASAN analysis center were obtained with the use of elevation cutoff angle equal zero (ftp://cddisa.gsfc.nasa.gov/pub/doris/products/sinex.series/inawd). But in this case a significant noise due to atmosphere refraction affects the results. Therefore for this study it was decided to recalculate all DORIS data for the period 1993.0–2009.0 with the elevation cutoff angle 15°. For this new solution [inawd06.snx] the next standards have been applied: gravity model – GGM01C (Tapley et al., 2003), atmospheric gravity – not applied, Ocean tides – CSR3, atmospheric density – DTM94 (Berger et al., 1998), drag parameterization – Cd/6hrs, troposphere mapping function – Lanyi (Lanyi, 1984). A complete description of the geophysical models used for [inawd06] solution and the estimation strategy can be found at: ftp://cddisa.gsfc.nasa.gov/pub/doris/products/sinex_series/inawd.

The weekly solutions of coordinates of all 71 DORIS ground sites and Earth Observations parameters (EOP) have been estimated with the use of new improved satellite surface models, submitted by CNES, and with measurement data of the satellites SPOT 2, SPOT 3, SPOT 4, SPOT 5, TOPEX, and ENVISAT. Data of JASON-1 were not used at all due to SAA (South Atlantic Anomaly) effect. This effect is related with extra sensitivity of the on-board receiver to radiation over South Atlantic Anomaly and gives meaningful DORIS residuals for POD (Precise Orbit Determination) estimation. Detailed investigation of SAA effect and developed correction model can be found in Willis et al. (2004) and Lemoine and Cardeville (2006), but this model is still in a testing phase by IDS Analysis Centres. It also should be noted that data of SPOT 4 for whole 1998 were rejected because of a systematic error that affects the Z-component geocenter estimation (Willis et al., 2006b).

3. Estimated parameters

After the transformation of the free-network solution into a well-defined reference frame (ITRF2005) stations coordinates of the DORIS network were estimated with the internal precision at the level of 5–40 mm for majority of the stations. As it was shown in Altamimi et al. (2006), a comparison of a cumulative DORIS solution, obtained by the use of weekly time-series for 12 years, with the similar International GNSS Service GPS solution at 37 collocated sites yields the same order of the WRMS: 8–12 mm in position and 2.7–3.8 mm/year in velocity.

DORIS positioning accuracy strongly depends on the number and constellation of satellites used in the solution (Gobinddass et al., 2009a). The systematic errors in the solutions may be caused by modeling errors in orbit determination.

In our solutions we estimated simultaneously X-pole, Y-pole coordinates and their rates once per day (four parameters per day). Mean square residuals of coordinates for the period 2000–2004 are estimated as 2.83 and 1.70 mas, respectively, with refer to IERS C04 solution (Gambis, 2006). It should be noted that better results could be obtained using DORIS polar motion estimates with fixed polar motion rates. The last results obtained with this strategy give polar motion estimates at the level 0.5 mas or better (Willis et al., 2009).

According to the IDS activity report (2006–2008) (http://ids-doris.org), comparison of orbit estimations, made by INA center and other IDS centers, showed...
inter-orbit consistency at the level of 1–2 cm in all directions.

4. Studies of the geocenter movements

Two sets of translation parameters, derived from DORIS weekly solutions of coordinates, calculated at the INA center [ina05wd01.geoc] and at the joint IGN/JPL center [ign07wd01.geoc] for the same time span 1993.0–2009.0, have been examined (Tatevian and Kuzin, 2009) with a view to study variations of the geocenter movements. We used also the IGN DORIS geocenter solution [ign07wd01.geoc] for comparison with the INA geocenter time series [ina05wd01.geoc], as they are very close in the way they were computed. But in the latest IGN geocenter solution [ign09wd01.geoc] the large artifact (solar radiation model) in the Z-component of geocenter was removed as discussed by (Gobinddass et al., 2009a,b). As a reference frame the ITRF2005 (exactly, long-term cumulative IGN solution: ign07d02) has been used for every week. For comparison geocenter time series, derived at the INA centre from the GPS daily coordinate solutions at JPL (ftp://side-show.jpl.nasa.gov/pub/ursr/mhb; Dong et al., 2003) for the time period 1992.5–2007.6 has been examined as well.

Two methods of spectral analysis of geocenter coordinates have been applied in our study.

4.1. The linear regression analysis

The linear regression analysis has been used in order to estimate linear trend, amplitudes, periods and phases of geocenter variations. For analysis the next approximation is used:

\[ J(t) = a_0 + b_0 t + A_0 \sin \left( \frac{2\pi}{T} (t - t_0) + \phi_0 \right), \]  

where \( A_0 \) is the amplitude of the signal; \( T \) the period of the signal (in years); \( \phi_0 \) the initial phase of the signal; \( a_0 \) the offset; \( b_0 \) the trend; \( t \) the time; \( t_0 \) the arbitrary initial time (we take \( t_0 = 1 \) January).

4.2. Dynamic regression modeling

The same time series of 16 years weekly geocenter coordinates \((X, Y, Z)\), (ftp://cdsjs.gsfc.nasa.gov/pub/doris/products/geoc/ina05wd01.geoc.Z) have been examined with the use of so-called method of adaptive Dynamic Regression Modeling (DRM) (Valeev, 1991; Valeev and Kurkina, 2006), which is realized by the special software AC DRM. This method includes: – a stochastic description of the time series and its studies with the – correlation, spectral and wavelet analysis; estimation and removal of the non-random trend component; – estimation of harmonic components. Unlike the linear regression analysis, the DRM method envisages the further iterative, step by step, regression analyses (Feder, 1991), of the non-random content of the de-trended series, obtained after first harmonics removing, aiming to avoid errors, caused by noise residuals and inter correlation between estimated harmonics and to find out the additional regularities. For that the appropriate adaptive mathematical description is selected from a set of applicable models, such as: auto regression – sliding mean, one of the GARCH model (Bollerslev et al., 1993), Kalman’s filter (Balakrishnan, 1988), martingale approximation. When at the appropriate step of analysis the mean square residual becomes unchangeable (without decreasing), process is completed, and residuals are analyzed on their correspondence to the basic requirements of the least square method. This approach allows to improve an accuracy of the estimated harmonics and to find out additional regular components of the examined time series. As a result of DRM-method the original time series is approximated by the complex mathematical model, which contains trend, periodical components and parameters of the dynamic regression model. In this paper we omitted the details of stochastic description and noise analysis of the evaluated time series.

5. Results of spectral analysis and prediction model

Annual and semi-annual geocenter variations derived by the linear regression analysis were estimated by least square method as 6.7 ± 0.2, 5.5 ± 0.1, 28.9 ± 1.1 mm for \( X, Y, Z \) components respectively. Geocenter annual variations, derived from the IGN/JPL solution [ign07wd01.geoc] for the same time period are 6.8 ± 0.2, 6.8 ± 0.1, 27.7 ± 0.8 mm in three components. There are rather good agreement between these DORIS solutions. Semi-annual amplitudes of the geocenter variations are also noticeable (1–34 mm in all components). The linear trend (−1.2, −0.1, −0.3 mm/year for \( X, Y, Z \) components) was found out of INA solution as well (Table 1). Annual and semi-annual amplitudes, estimated by GPS (JPL) data are lower (0.21–2.1 mm), and values of linear trend are almost negligible. Amplitudes and phases of the evaluated annual and semiannual variations of the geocenter components \( X, Y, Z \) are presented in Table 2.

Analysis of the same INA time series of the geocenter motions with the method of adaptive Dynamic Regression Modeling allowed to find out several additional harmonics with periods of ~2, 3, 5 years and with valuable amplitudes. For \( X \)-component seven harmonics without inter-correlation and with mean square residual \( \sigma_A = 4.8 \) mm

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Linear trend of the geocenter coordinates ((X, Y, Z)), estimated by different solutions.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solution</td>
<td>( X ) (mm/y)</td>
</tr>
<tr>
<td>DORIS/INA (ina05wd01.geoc)</td>
<td>−1.19 ± 0.07</td>
</tr>
<tr>
<td>DORIS/IGN-JPL (ign07wd01.geoc)</td>
<td>0.29 ± 0.04</td>
</tr>
<tr>
<td>GPS/JPL</td>
<td>−0.06 ± 0.05</td>
</tr>
</tbody>
</table>
were derived. Among them four harmonics with periods 25, 53, 169, 282 weeks have noticeable amplitudes: 2.1, 5.9, 3.2, 2.9 mm respectively. From the set of \( Y \)-component three noticeable harmonics were derived with periods 18, 53, 282 weeks and amplitudes 1.7, 6.1, 2.5 mm respectively. The harmonics with periods 17, 53, 106, 141, 282 weeks were derived for \( Z \) component with \( r_D = 38.4 \) mm. Amplitudes of these components are higher, than for \( X \) and \( Y \), and equal 11.0, 24.2, 9.6, 11.2, 11.2 mm respectively.

Amplitudes of annual and semi-annual variations of geocenter, derived with the use of two different methods of spectral analysis are in very good agreement between each other. The evaluated annual amplitudes are of the order of 5–7 mm for \( X \) and \( Y \) components and 25–28 mm for \( Z \) component. Semi-annual amplitudes of the geocenter variations are also noticeable in all components.

The results of the Dynamic Regression Modeling have been used for developing of the complex mathematical model for \( X \), \( Y \) and \( Z \) components of the geocenter movements for the span 1993.0–2009.0. The equations of this model presented in Appendix A. These equations consist of the trend component, poly-harmonic model, GARCH (1, 1) model (Bollerslev et al., 1993) and of Kalman filter.

6. Experimental estimate of the mathematical model of the geocenter movement

With a view of estimating an accuracy and probability of the developed complex mathematical model of the geocenter movement we used the same DRM approach for evaluation of the shortcut model, covering only 15 years time span (1993.0–2008.0). With this model a forecasting of the weekly geocenter positions for the next, 2008, year has been performed. The results are presented by the graphs (Figs. 1–3) for three components (\( X, Y, Z \)) of geocenter variations. The values of weekly geocenter coordinates, evaluated at INASAN center with the use of DORIS data for the 2008 year (ina05wd01.geoc.Z), are regarded as “observable” (black line). Values, simulated by the model, are plotted by the dotted line.

Fig. 1. Graph of the geocenter (\( X \)-component) movement forecast (dotted line) for 2008 and “observable” (black line) positions, estimated by DORIS data analyses (on the abscissa – time in weeks, on the ordinate – geocenter shift in mm).

Table 2
Amplitudes and phases of the evaluated annual and semiannual variations of the geocenter components \( X, Y, Z \).

<table>
<thead>
<tr>
<th>Component</th>
<th>Time interval (years)</th>
<th>Annual</th>
<th>Semi-annual</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_X )</td>
<td>DORIS (INASAN)</td>
<td>1993.0–2009.0</td>
<td>6.7 ± 0.2</td>
</tr>
<tr>
<td>( T_Y )</td>
<td>DORIS (INASAN)</td>
<td>1993.0–2009.0</td>
<td>5.5 ± 0.1</td>
</tr>
<tr>
<td>( T_Z )</td>
<td>DORIS (INASAN)</td>
<td>1993.0–2009.0</td>
<td>28.9 ± 1.1</td>
</tr>
<tr>
<td>( T_X )</td>
<td>DORIS (IGN/JPL)</td>
<td>1993.0–2009.0</td>
<td>6.8 ± 0.2</td>
</tr>
<tr>
<td>( T_Y )</td>
<td>DORIS (IGN/JPL)</td>
<td>1993.0–2009.0</td>
<td>6.8 ± 0.1</td>
</tr>
<tr>
<td>( T_Z )</td>
<td>DORIS (IGN/JPL)</td>
<td>1993.0–2009.0</td>
<td>0.41 ± 0.02</td>
</tr>
<tr>
<td>( T_X )</td>
<td>GPS (JPL)</td>
<td>1992.5–2007.6</td>
<td>0.58 ± 0.03</td>
</tr>
<tr>
<td>( T_Y )</td>
<td>GPS (JPL)</td>
<td>1992.5–2007.6</td>
<td>0.58 ± 0.03</td>
</tr>
<tr>
<td>( T_Z )</td>
<td>GPS (JPL)</td>
<td>1992.5–2007.6</td>
<td>0.58 ± 0.03</td>
</tr>
</tbody>
</table>
from the measurements) dynamics of geocenter movements. Summarizing the results of these experimental calculations we can conclude, that correlation coefficient between simulated data and “observable” values for the annual time interval for $X$ component is 0.786, and RMS is 7.01 mm. For the first 10 weeks of the 2008 year the simulated data are most consistent with the “observations”, and in this case correlation coefficient is 0.852 and RMS equals to 2.24 mm. For $Y$ component the correlation coefficient between forecasting values and “observations” on the annual interval equals to 0.766, and RMS is 7.52 mm. For the time period 10 weeks the correlation is 0.949 and RMS is equal to 2.26 mm. A correlation between forecasting and “observable” variations of $Z$ component is 0.802 and 0.815 for the annual and 10 weeks periods respectively. The mean square residuals are 29.13 and 12.92 mm.

More significant error in $Z$-component, corresponding to a translation of the Earth along its rotation axis, may be partly explained by large systematic errors in orbital calculation strategy of some of the DORIS satellites. In the latest studies (Gobinddass et al., 2009b) was shown that better handling of solar pressure radiation effects on SPOT-2 and TOPEX satellites significantly improves the measurement noise of the $Z$-geocenter component and accordingly, amplitudes of the annual signal decrease from 35 to 6 mm.
7. Conclusion

A reprocessing of the DORIS data for the period of 1993.0–2009.0 has been performed aiming to obtain a unified coordinated solution of the IDS analysis centers for the developing of the new version of the Terrestrial Reference Frame – ITRF2008. Two sets of translation parameters, derived from DORIS weekly solutions of coordinates, calculated at the INA center [ina05wd01.geoc] and at the joint IGN/JPL center [ign07wd01.geoc] for the same time span, were analyzed. The DORIS system was chosen because of almost equal distribution of DORIS ground beacons over the globe, that is, on our opinion, very important for the geocenter position determination. At the same time amplitudes of annual and semi annual geocenter variations, evaluated by the analysis of DORIS data, are significantly 2–3 times as much as those, derived from the GPS and SLR measurements, mainly in the geocenter Z-component. (Crétaux et al., 2002; Tatevian et al., 2004; Lavalle et al., 2006). It was shown by several authors (Gobinddass et al., 2009a), that systematic errors in geocenter estimation by the DORIS measurements are satellite dependent, and improved satellites orbital modeling has to be applied to avoid the discrepancies between different geocenter solutions. Nevertheless we assume that general behavior of the geocenter motion, estimated with the use of multi-years DORIS time series, more or less coincide with its real dynamics.

The first attempt to develop a mathematical model of the geocenter motion has been made with the use of Dynamic Regression Modeling approach for spectral analysis of the long set (16 years) of geocenter coordinates, estimated by DORIS measurements.

It was shown, that the obtained model may be used for prediction of the further geocenter motion during the next 10 weeks with the RMS: 2.24 mm (X), 2.26 mm (Y) and 12.92 mm (Z). Further investigations in this direction will be performed with different types of measurements, such as SLR and GPS, and with the improved orbital modeling of the DORIS satellites.

Acknowledgments

We thank the Associate Editor Pascal Willis (IGN, France) and the reviewers for their thorough revision and advices that helped to improve the manuscript. The work was partly funded by Russian Basic Science Foundation. We would like to thank the IDS, ILRS and IGS data centers, which make data of satellite measurements available.

Appendix A

As a result of DRM-method the original time series (1993–2009) are approximated by the complex mathematical model, which contains trend, periodical components and parameters of the dynamic regression model: GARCH (1.1) and Kalman filter.

Eqns. (A1)–(A3) show three components of the model, where $\epsilon_{ds}(t), \epsilon_{dy}(t), \epsilon_{dz}(t)$ are noise components

\[
y_{ds}(t) = 7.798 - 0.0686 \cdot t + 0.0000508 \cdot t^2 \\
+ 0.862 \cdot \sin \left( \frac{2\pi t}{8} - 11.636 \right) \\
+ 2.062 \cdot \sin \left( \frac{2\pi t}{25} + 96.81 \right) \\
+ 5.924 \cdot \sin \left( \frac{2\pi t}{53} + 87.364 \right) \\
+ 1.48 \cdot \sin \left( \frac{2\pi t}{70} + 127.22 \right) \\
+ 1.97 \cdot \sin \left( \frac{2\pi t}{121} + 190.43 \right) \\
+ 3.168 \cdot \sin \left( \frac{2\pi t}{169} + 173.25 \right) \\
+ 2.906 \cdot \sin \left( \frac{2\pi t}{282} + 112.05 \right) + 0.375 \cdot y_{2,ds}(t-1) \\
+ 0.191 \cdot y_{2,ds}(t - 2) - 0.005 \cdot y_{3,ds}(t - 3) \\
- 0.036 \cdot y_{2,ds}(t - 4) + 0.059 \cdot y_{2,ds}(t - 5) \\
+ 0.129 \cdot y_{2,ds}(t - 6) + 0.107 \cdot y_{3,ds}(t - 1) + \epsilon_{ds}(t) \\
\]

(A1)

\[
y_{dy}(t) = 11.151 - 0.0077 \cdot t + 0.000067 \cdot t^2 \\
+ 1.142 \cdot \sin \left( \frac{2\pi t}{10} + 164.74 \right) \\
+ 1.05 \cdot \sin \left( \frac{2\pi t}{18} + 222.08 \right) \\
+ 6.146 \cdot \sin \left( \frac{2\pi t}{53} - 64.08 \right) \\
+ 2.47 \cdot \sin \left( \frac{2\pi t}{282} - 54.726 \right) + 0.132 \cdot y_{2,dy}(t - 1) \\
+ 0.162 \cdot y_{2,dy}(t - 2) + 0.017 \cdot y_{2,dy}(t - 3) \\
+ 0.093 \cdot y_{2,dy}(t - 4) + 0.023 \cdot y_{2,dy}(t - 5) \\
+ 0.121 \cdot y_{2,dy}(t - 6) + 0.197 \cdot y_{3,dy}(t - 1) + \epsilon_{dy}(t) \\
\]

(A2)

\[
y_{dz}(t) = -54.003 + 0.297 \cdot t - 0.000344 \cdot t^2 \\
+ 8.529 \cdot \sin \left( \frac{2\pi t}{10} + 243.69 \right) \\
+ 11.074 \cdot \sin \left( \frac{2\pi t}{17} + 125.67 \right) \\
+ 24.177 \cdot \sin \left( \frac{2\pi t}{53} + 268.71 \right) \\
+ 9.645 \cdot \sin \left( \frac{2\pi t}{106} + 29.597 \right) \\
+ 11.235 \cdot \sin \left( \frac{2\pi t}{141} - 44.677 \right) + 0.412 \cdot y_{2,dz}(t - 1) \\
+ 0.162 \cdot y_{2,dz}(t - 2) + 0.009 \cdot y_{2,dz}(t - 3) \\
- 0.002 \cdot y_{2,dz}(t - 4) + 0.019 \cdot y_{2,dz}(t - 5) \\
+ 0.113 \cdot y_{2,dz}(t - 6) - 0.304 \cdot y_{3,dz}(t - 1) + \epsilon_{dz}(t) \\
\]

(A3)
References