Empirical Modeling of the Retrograde Free Core Nutation (Technical Note)

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1 Introduction

This note presents the derivation of an empirical model for the mantle oscillation associated with the retrograde free core nutation (RFCN).

2 Least-Squares Model

I use the daily combined series IERS EOP 05 C 04 (referred to as C04 in the following) from January 1st, 1984 computed at the IERS Earth Orientation Center at the Paris Observatory (Gambis & Bizouard 2007). C04 series provides daily values for celestial pole offsets dX and dY referred to the MHB expansion. I fix the period of the free motion to the estimated value in the MHB work, let -430.21 days in a spaced-fixed frame of reference, and I consider only changes in the phase. Moreover, the space motion of the figure axis due to the RFCN is considered as circular, ignoring any possible asymmetry in the distribution of mass in the core.

The computation is based on a weighted least-squares fit of a circular term plus a constant to the complex-valued quantity $dX + \mathrm{i}\, dY$:

$$dX + \mathrm{i}\, dY = A\mathrm{e}^{\mathrm{i}\,\sigma t} + X_0 + \mathrm{i}\, Y_0, \tag{1}$$

where A is the complex amplitude, σ the RFCN frequency, and t is the time measured from J2000.0. This leads to:

$$\begin{aligned} dX &= A_c \cos\sigma t - A_s \sin\sigma t + X_0, \\ dY &= A_c \sin\sigma t + A_s \cos\sigma t + Y_0, \end{aligned} \tag{2}$$

allowing the estimation of 4 parameters: A_c and A_s and the constant offsets X_0 and Y_0. The offsets account for the long-term variations appearing in the nutation residuals and are not physically related to the core nutation. The contribution of the RFCN only to the celestial pole offsets is given by:

$$\begin{aligned} X_{\mathrm{RFCN}} &= X_s \sin\sigma t + X_c \cos\sigma t, \\ Y_{\mathrm{RFCN}} &= Y_s \sin\sigma t + Y_c \cos\sigma t, \end{aligned} \tag{3}$$

where:

$$X_s = Y_c = A_s, \quad X_c = -Y_s = A_c. \tag{4}$$

To account for the time variability of the amplitude and the phase, the estimates are done over a N_L-length sliding window displaced by N_D. The tabulated epoch for each window is the middle date.

3 Size and Displacement of the Sliding Window

To determine the most suitable values for N_L and N_D, I proceed with a synthetic data consisting of a damped, free motion (taken from any previous RFCN empirical model, for instance the one of Malkin & Terentev 2007 derived by a wavelet analysis of VLBI data) on which I add a gaussian noise of variance

Table 1: Coefficients of the amplitude of the free motion as fitted on July 2007.

year	mjd	X_c, $-Y_s$ μas	X_s, Y_c μas	$\pm$ μas
1984	45700	78.87	-125.13	18.93
1985	46066	-94.06	-128.40	12.32
1986	46431	-238.06	-154.23	14.25
1987	46796	-262.91	-152.30	13.65
1988	47161	-201.75	-27.67	14.20
1989	47527	-191.80	-87.82	11.94
1990	47892	-201.02	-60.12	9.18
1991	48257	-147.87	26.27	7.92
1992	48622	-118.88	40.70	6.51
1993	48988	-111.93	9.64	7.51
1994	49353	-120.51	34.38	6.99
1995	49718	-123.15	39.28	7.67
1996	50083	-143.22	17.15	7.41
1997	50449	-127.36	49.06	7.50
1998	50814	-82.19	30.73	7.63
1999	51179	-19.96	-35.48	6.33
2000	51544	29.07	-111.01	5.95
2001	51910	86.98	-123.91	7.36
2002	52275	80.07	-97.29	7.13
2003	52640	109.90	-54.14	4.65
2004	53005	131.46	-10.18	4.55
2005	53371	157.76	-15.35	3.23
2006	53736	160.41	-14.18	3.08
2007	54101	148.90	34.30	4.31

$\sim$200 μas. The fitted free motion is then compared to the original (noise-free) signal in terms of rms and correlation coefficient. The procedure is repeated a thousand times to get a reliable statistics and for $200 < N_L < 1,000$ days and $200 < N_D < N_L$ days. Fig. 1 reports on the obtained correlation and rms for the X-component (the results for the Y-component are similar). It appears that the best fit (high correlation coefficient and low rms) is generally obtained for a window length around or larger than 2 years. The correlation coefficient and the rms lie in the red and green/blue regions, respectively. For my model, I choose $N_L = 2$ years (i.e., around 730 days, fluctuating to account for leap years) and $N_D = 1$ year. Thus I can get one estimate per year every January 1st. Note that the rms for this couple of values is around 40 μas. It is about three times smaller than the uncertainty on VLBI data and it is significantly higher than the formal error of the least-squares fit (see next section). It constitutes a pessimistic evaluation of the error on the model. Also, one has to note that this rms results from an estimation on the full time span, although the 'local' amplitude of the noise varies significantly between the early VLBI and after 2000.

4 Final Results and Prediction Error

Fig. 2 shows the superimposition of the C04 data set and of the fitted free motion. As an example, this is the result of a fit done on the C04 data set running till July 2007. The numerical values of the yearly amplitudes are reported in Tab. 1. It can be noticed that the formal error on these amplitudes varies between 20 μas in the early years down to 3–5 μas for the most recent years. As already mentioned, the reader must keep in mind that a more realistic error estimated through statistical tests might replace these formal errors.

A Fourier spectrum is shown on Fig. 3. The spectral peak associated with the free motion is centered around -0.84 cycle per year and appears as a broad peak (and even as a double peak) because of phase

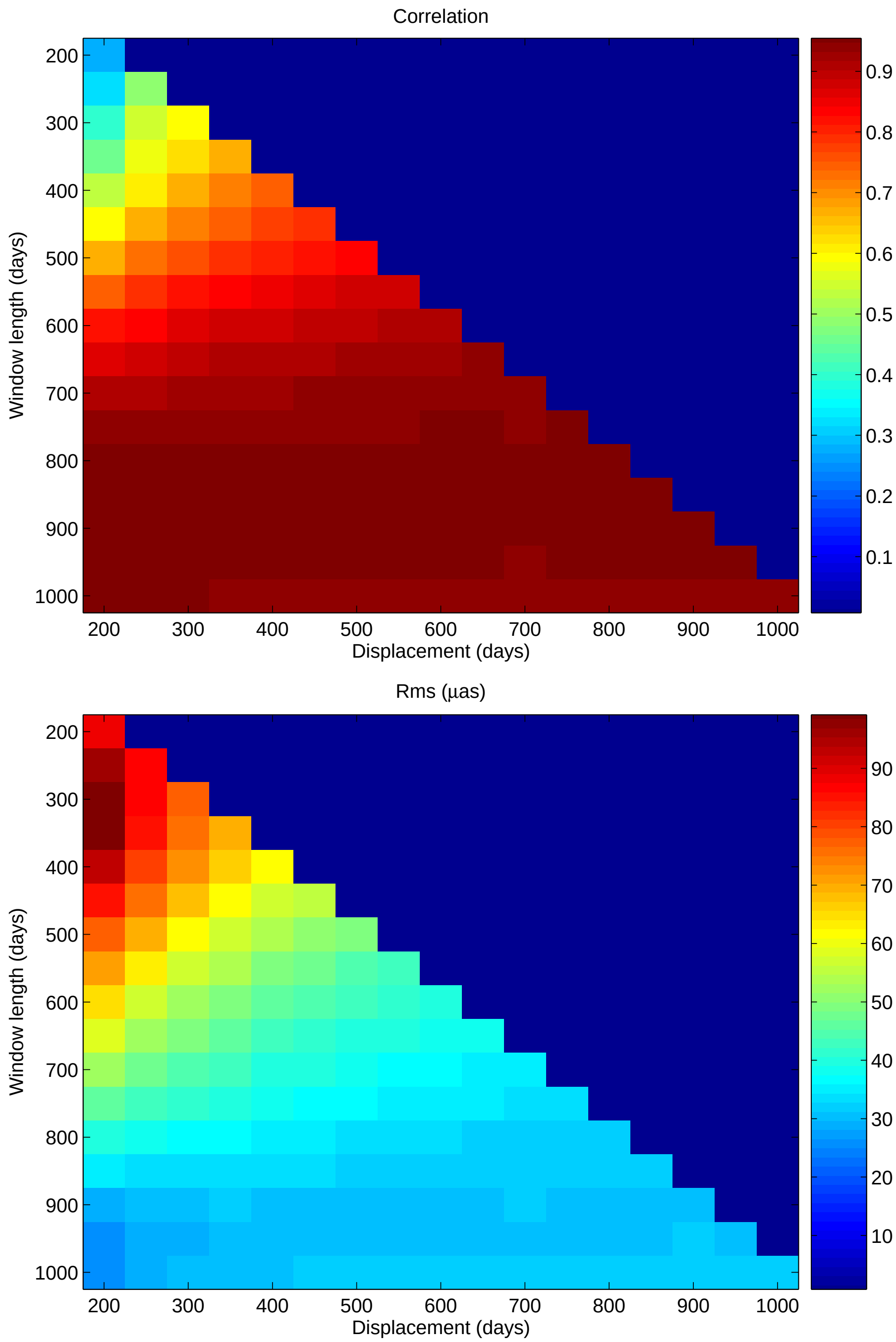

Figure 1: Simulation for testing the sensitivity of the least-squares fitting algorithm to N_L and N_D.

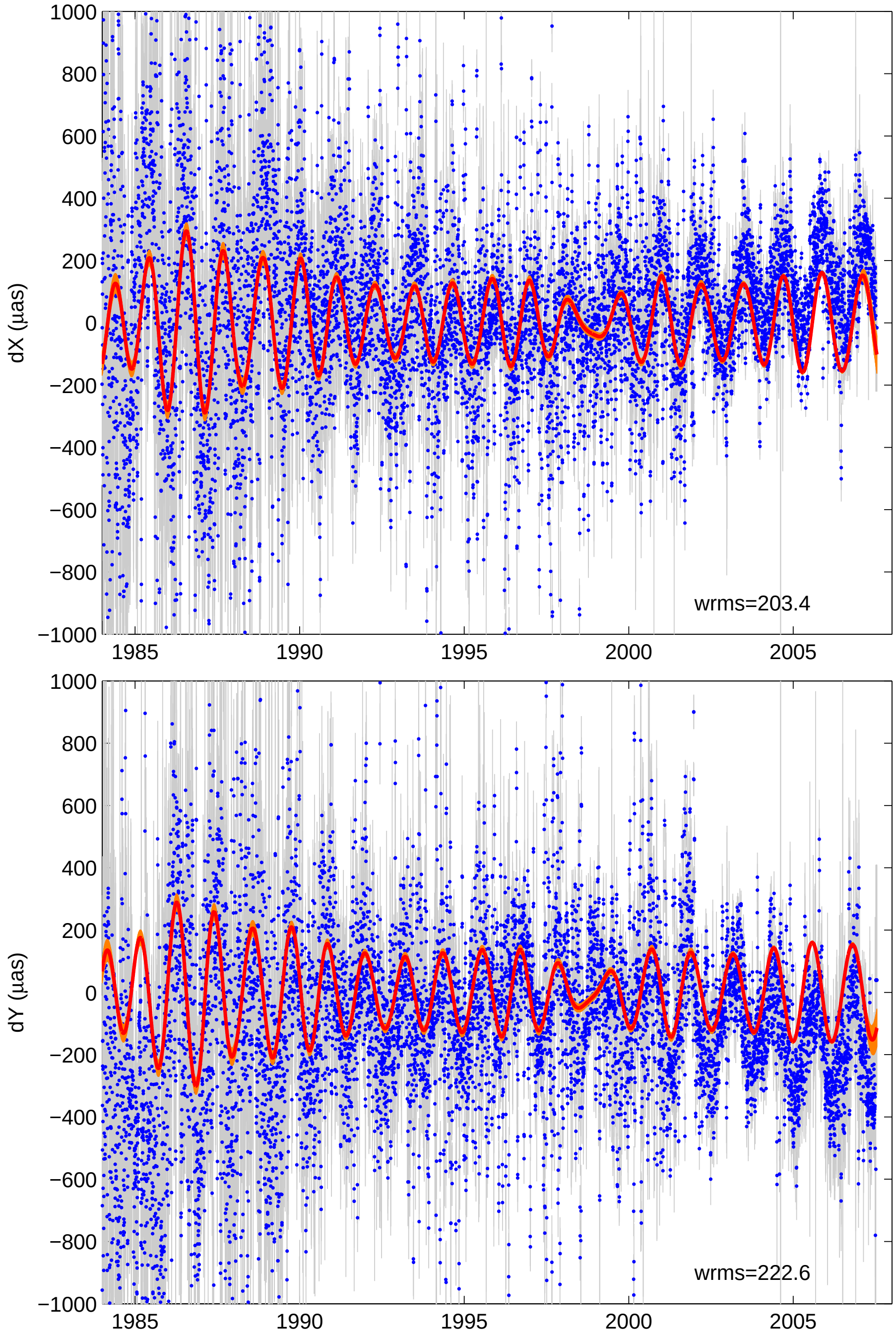

Figure 2: C04 data set and free motion as fitted on July 2007.

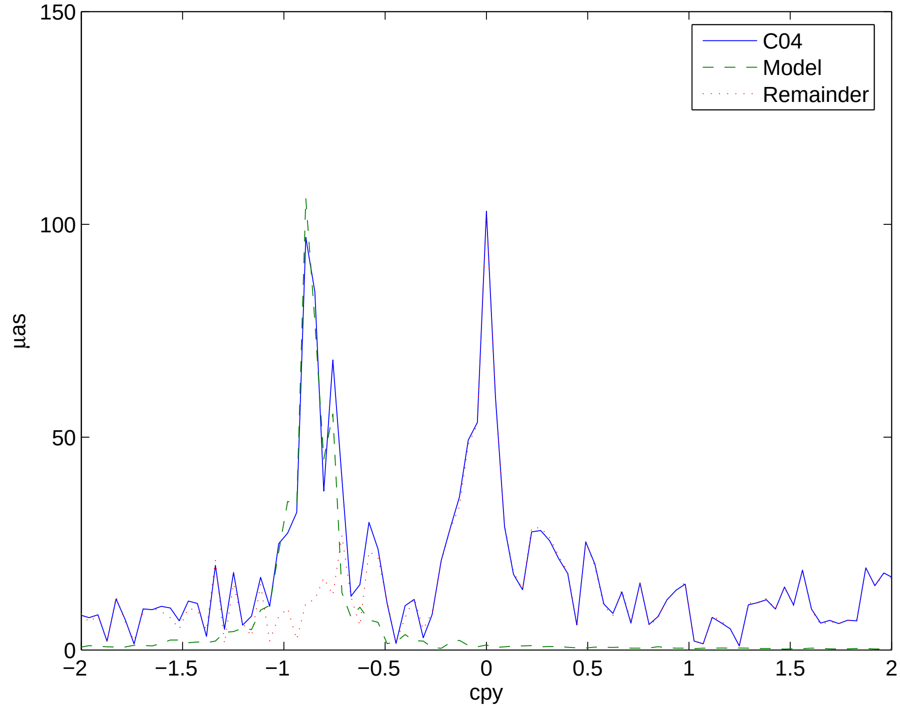

Figure 3: Fourier spectrum of the C04, the free motion and the difference.

variations, detected as variations of the period by the Fourier transform (see Vondrák et al. 2005). The peak showing up around the zero-frequency is the signature of long-term time variations in the nutation offsets that are not treated in this modeling. After removing the free motion from the C04 data set, the remaining power is statistically not significant at the RFCN frequency.

The mean prediction error has been estimated through the average of a thousand predictions over past time intervals (see Fig. 4). It appears that a second order polynomial fits very well the prediction error, with a postfit rms of 0.2 μas, whereas a linear regression leads to a postfit rms of 3 μas. However, since the formal error on the amplitude of the free motion remains larger than 3 μas, the choice of the linear regression to account for the degradation of the error with time seems sufficient. For the routine implementation, I adopt the value of 0.1325 μas/day for the degradation in forward or backward predictive mode starting from any given epoch.

5 Availability of the Results and Updates

A web site providing the user with the model coefficient values, plots, and ASCII files giving time series of free motion and a 1-year prediction is made available on the IERS web site at:

`ftp://hpiers.obspm.fr/eop-pc/models/fcn/index.html`

A Fortran 77 subroutine is also available on the web site. Given a date (in modified julian day), it returns the values of X_{RFCN} and Y_{RFCN} in μas as well as their formal errors. The amplitude variation between two nodes (epochs) is modeled by a piecewise linear function. This routine has been used for producing the free motion time series shown on Fig. 3. Use the syntax below:

```
c INPUT

double precision mjd      ! modified julian date

c OUTPUT
```

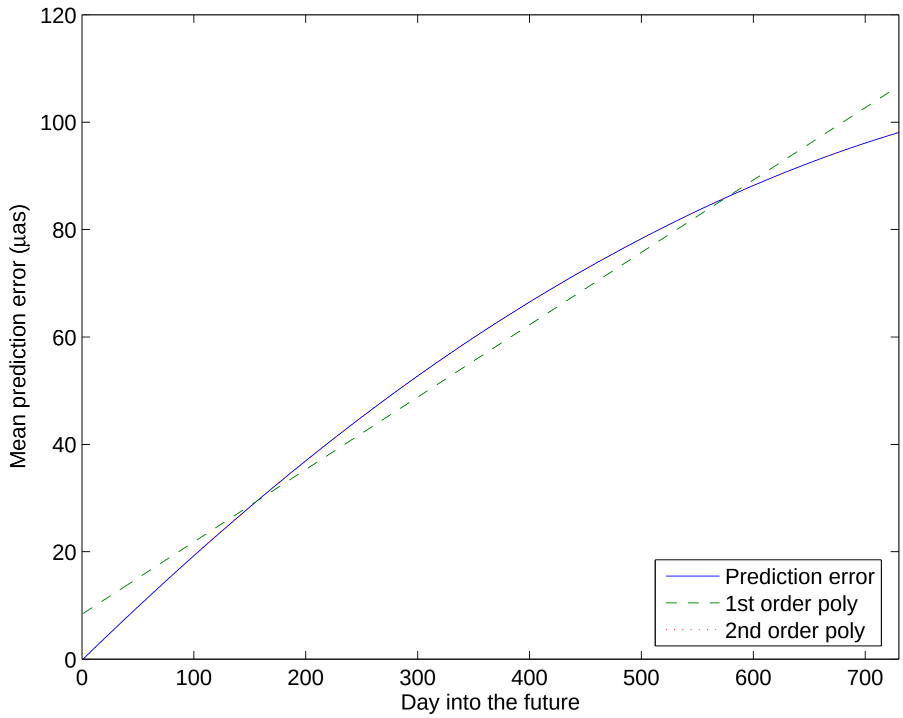

Figure 4: Mean prediction error brought by the fitted model of free motion and polynomial fits.

```
double precision X,Y       ! contribution on X and Y (microas)
double precision dX,dY     ! error on the contributions (microas)

c SUBROUTINE CALL

call fcnnut(mjd,X,Y,dX,dY)
```

The update frequency can be twice a year. Unless the C04 data set is strongly modified (e.g., due a complete reanalysis after changing the IERS combination strategy), the amplitudes for past years will remain the same or very close. Only the coefficient relative to the present year can be affected significantly.

References

[1] Gambis, D. & Bizouard, C. 2007, IERS EOP 05 C 04 data computed at the IERS Earth Orientation Center, Paris Observatory, France, and made available at http://hpiers.obspm.fr/iers/eop/eopc04_05/eopc04_IAU2000.63-now

[2] Malkin, Z., & Terentev, D. 2007, arXiv:physics/0702152

[3] Mathews, P.M., Herring, T.A., & Buffett, B.A. 2002, J. Geophys. Res., 107(B4), doi:10.1029/2001JB000390

[4] Vondrák, J., Weber, R., & Ron, C. 2005, A&A, 444, 297