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Short Note: Conventions on thermal expansion modelling of radio telescopes for geodetic and astrometric VLBI

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Abstract Thermal expansion of radio telescopes has long been recognized as an effect which cannot be neglected in geodetic and astrometric VLBI data analysis if mm accuracy is desired. In this publication, the author documents the conventions which are being set by the International VLBI Service for Geodesy and Astrometry (IVS) for a consistent modelling of this effect in its routine product generation. For the largest telescopes the annual cycle of thermal expansion may change the height of the VLBI reference point by as much as 20 mm. However, for telescopes which are used in present-day IVS operations, the variations rather range from 4 to 6 mm.

Keywords Geodetic VLBI · Radio telescopes · Thermal expansion

1 Introduction

The modelling of thermal expansion effects of radio telescopes in geodetic and astrometric VLBI data analyses has been addressed in the literature already for some time (e.g. Nothnagel et al. 1995, Elgered et al. 1995, Haas et al. 1999, Skurikhina 2001, Wresnik et al. 2007). However, for a consistent use of these models in the context of operational data analysis for the determination of earth orientation parameters or for computations with respect to the International Terrestrial Reference Frame (ITRF) (Altamimi et al. 2007) a number of issues had not been settled yet. In addition, an improved modelling is being made available in this paper which also addresses a few parts missing in previous publications. Therefore, this paper is supposed to serve as a reference for analysts in an operational environment, e.g. within the International

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VLBI Service for Geodesy and Astrometry (IVS) (Schlüter et al. 2002), and will now permit a consistent use of a conventional model for the correction of VLBI observations for thermal expansion effects.

When considering the modelling of thermal expansion of radio telescopes in the framework of the Conventions of the International Earth Rotation and Reference Systems Service (IERS Conventions) (McCarthy and Petit 2003), it belongs to the category of "Displacement of reference points of instruments". Up to now, thermal expansion has been covered rudimentarily in the IERS Conventions but it had been proposed that the services of the International Association of Geodesy (IAG) like the IVS should maintain their own conventions for purely technique-related issues. Through a series of decisions, the IVS has now taken over the responsibility for thermal expansion effects, in particular of radio telescopes (Petit, pers. comm.).

2 VLBI Antenna Thermal Deformation

2.1 General

VLBI telescopes as any other steel or concrete constructions are distorted by time-dependent temperature effects. As a consequence the VLBI reference points are displaced with respect to a mean position. In the absence of a more refined model which includes asymmetric expansion and distortion effects, it is assumed that the telecope structures and their components expand or shrink linearly with temperature in a symmetric way only. In the literature, the expansion coefficients for steel including stainless steel range from 10×10^{-6} to 16×10^{-6} per °C with iron having 12.2×10^{-6} (e.g. Tipler and Mosca 2007). Without knowing the exact material of any individual telescope, it is appropriate to assume a global expansion coefficient of 12×10^{-6} for the steel/iron parts of the telescopes.

A similar situation applies to concrete where the type of stone imbedded determines the expansion coefficient which ranges from 6×10^{-6} to 14×10^{-6} per °C. A mean of 10×10^{-6} per °C should be applied if no other information is available.

During VLBI sessions, air temperature is one of the in-situ measurements that are recorded at VLBI sites. The time delay between the change in the surrounding air temperature and the expansion of the telescope structure depends on the material of the telescope. Measurements yielded values of 2 hours for a steel telescope structure (Nothnagel et al. 1995) and of 6 hours for a concrete telescope structure (Elgered and Carlsson 1995).

Variations in the position of the VLBI reference point have to be referred to a mean position and, in the context of thermal expansion, thus, to a mean temperature. A single global mean temperature like 20° C which is often used in civil engineering would not serve well the global distribution of telescopes between the arctics and the tropics. Therefore, a decision on a reference or mean temperature for each site individually had to be made on the basis of two requirements: First, the choices should be easy to implement and should permit a consistent computation for all telescopes with observations in the IVS archives as well as for any new station to take up operations. Second, the choices should not affect any realisations of the International Terrestrial Reference Frame (ITRF) and especially its scale. Therefore, a proposal was made by Wresnik et al. (2007) to use the GPT model (Boehm et al. 2007). GPT is a global temperature and pressure model based on a spherical harmonic expansion of degree and order nine with an annual periodicity. April 28 or MJD 44357.3125 is zero phase of the annual cycle.

The fact that GPT computed for the April 28 epoch provides a good representation of the global mean temperature field was tested by Boehm et al. (2008). To verify that GPT meets also the second requirement, tests have been made to study whether the implementation of the GPT model for the computations of reference temperatures would affect the ITRF. The result was that only a few telescopes show height deviations of more than 0.1 mm with most of them being of little importance because they have only been in use for a limited time period in the past (Heinkelmann et al. 2007). On this basis, the IVS accepted the GPT model at epoch April 28 for the computations of the reference temperatures.

2.2 Radio Telescope Construction Elements

Radio telescope focus concepts are mainly realized with a single reflection and the feed horn located in primary focus of the paraboloid or with a second reflection by a subreflector using a secondary focal point. In the latter case, most VLBI telescopes are of Cassegrain type with a few exceptions employing Gregory focus (e.g. Effelsberg, Germany).

The telescope dishes are mounted in different ways. Today, the most common type is the azimuth-elevation (alt-azimuth) mount where the fixed (primary) axis is oriented along the local vertical (Fig. 1.1 and 1.2). Polar mounts (Fig. 1.3) have their primary axis oriented parallel to the earth's rotation axis permitting tracking of celestial objects by single-axis rotations. Here, the primary axis is called the polar or hour-angle axis. A special case of this type is the historical Richmond antenna in Florida, USA, which was relocated from a position near Washington D.C. (39.06° North) to a different latitude (25.6°) complicating the otherwise simple geometrical relationships. Finally, the X/Y mounts (Fig. 1.4) have their fixed axes oriented parallel to the local horizontal plane oriented either North-South (e.g. Gilmore Creek, Alaska, USA) or East-West (e.g. Hobart, Tasmania, Australia) permitting a better tracking of celestial objects through the local zenith.

The moving (secondary) axes are usually designed to be perpendicular to the fixed axes. Most radio telescopes are constructed in a way that the secondary axis does not intersect the fixed primary axis, i.e. the azimuth axis or polar axis, leading to a so-called axis offset (AO in figures 1.1. - 1.4). Polar mount telescopes as well as X/Y mount antennas always have secondary axes which lie above the primary axes and, thus, these axis offsets have to be considered with a positive sign. In contrast to this, the secondary axis of an alt-azimuth telescope may lie behind the azimuth axis (Fig. 1.2) and in such a case the sign of the axis offset is negative.

Currently, one exception exists for the general assumption of perpendicular axes which is the telescope of GARS O'Higgins, Antarctica, with the secondary axis tilted by 45°. However, since the primary and secondary axes intersect with zero axis offset, it can be modelled as a standard azimuth-elevation antenna.

Figures 1.1. - 1.4 serve as a reference for the description of the construction elements. The height of the concrete foundation is denoted by h_f, the height of the antenna pillar by h_p, the height of the vertex by h_v, the height of the subreflector by h_s and the axis offset between the two axes by AO. A list of antenna construction elements and other necessary parameters is maintained under http://vlbi.geod.uni-bonn.de/IVS-AC/Conventions.

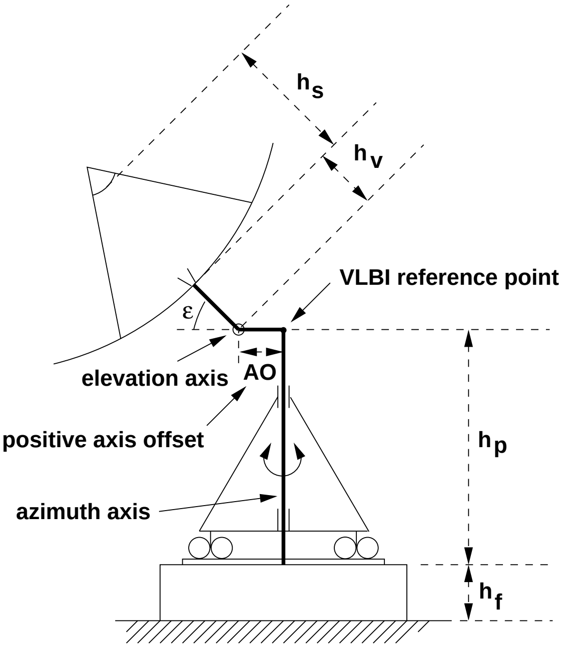

Fig. 1.1 Alt-azimuth telescope mount with positive axis offset.

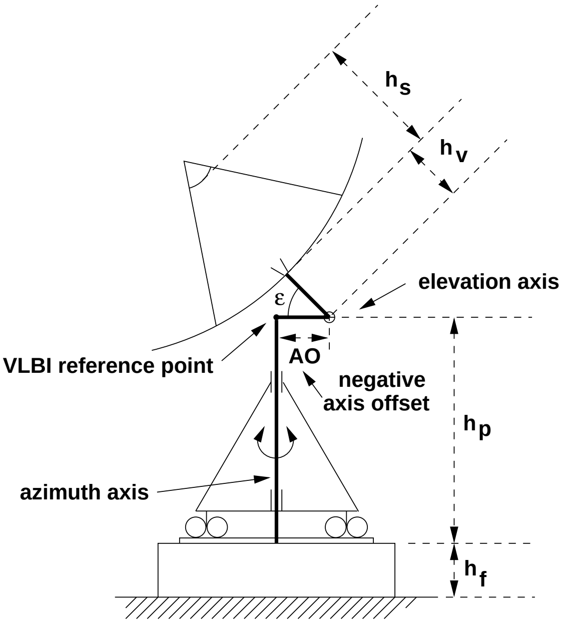

Fig. 1.2 Alt-azimuth telescope mount with negative axis offset.

2.3 Delay Corrections

The time delay caused by thermal expansion of the telescope's construction elements is modelled according to the following formulas which are partly collected from Nothnagel et al. (1995), Haas et al. (1999) and Skurikhina (2001). The latter contained a general sign error and a duplicate application of the axis offset for polar mounts which are corrected here.

In the equations below, c is the speed of light [m/s], γ_f and γ_a are the expansion coefficents $[1/^\circ C]$ for the foundation and for the antenna construction elements (mostly steel), respectively, and h_f, h_p, h_v, and h_s are the dimensions of the telescopes [m] while AO [m] is the axis offset between primary and secondary axis. δ is the declination of the radio source being observed while azimuth and elevation of the observed radio source are denoted by α and ε. The elevation may be corrected for refractivity but the effect on the thermal expansion modelling is negligible. F_a is the antenna focus factor which reflects the effective signal path between the main reflector and the feed horn. For prime focus antennas F_a is 0.9 and for secondary focus antennas F_a is 1.8 (Otoshi and Young 1982).

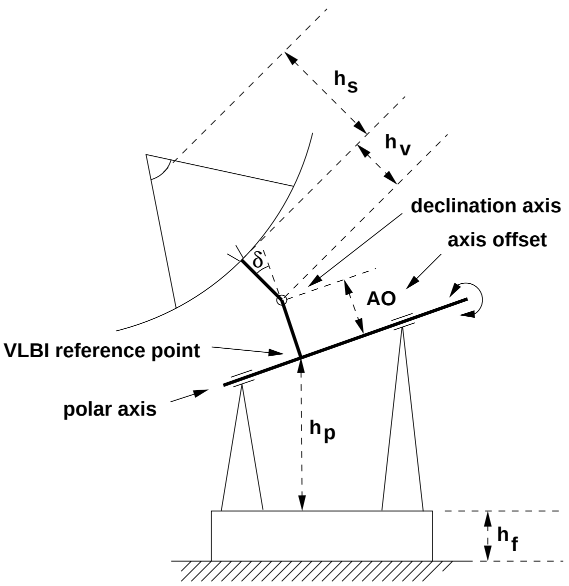

Fig. 1.3 Polar telescope mount.

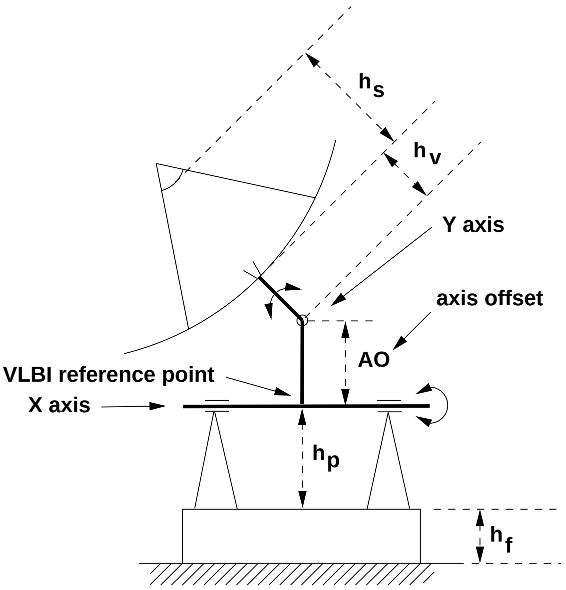

Fig. 1.4 X/Y telescope mount.

The temperatures of the telescope structure elements are normally not available unless a dense set of temperature sensors is mounted. Therefore, the current model for most antennas has to be based on the surrounding air temperature denoted by T and the respective time lags for the expansion taking effect (e.g. 2 hours for steel and 6 hours for concrete). T_0 is the reference (air) temperature with t being the observation epoch and $\varDelta t_a$ (antenna) and $\varDelta t_f$ (foundation) denoting the time lags.

Except of the one case where the elevation axis of an alt-azimuth antenna lies behind the azimuth axis (Fig. 1.2), the antenna axis offset brings the receiver closer to the incoming wavefront. A wavefront, thus, arrives earlier at the receiver by the axis offset contribution $\varDelta\tau_{axis}$ which depends on the unit vector in source direction $\mathbf{s}$ and the unit vector in the direction of the fixed axis $\mathbf{f}$ (Skurikhina 2001 corrected):

$$\varDelta\tau_{axis} = \frac{1}{c} AO \cdot \sqrt{1 - (\mathbf{s} \cdot \mathbf{f})^2}. \tag{1}$$

This term changes for the different antenna mounts. With the axis offset term expressed w.r.t. azimuth, elevation or declination, the contribution of the thermal expansion effects on the time of arrival of a wavefront relative to the VLBI reference point is as follows:

a) For alt-azimuth mounts

$$\begin{aligned}\varDelta\tau_{therm.i} = \frac{1}{c} \cdot \Big[&\gamma_f \cdot (T(t - \varDelta t_f) - T_0) \cdot (h_f \cdot \sin\varepsilon) \\ &+ \gamma_a \cdot (T(t - \varDelta t_a) - T_0) \cdot (h_p \cdot \sin\varepsilon + AO \cdot \cos\varepsilon + h_v - F_a \cdot h_s) \Big].\end{aligned} \tag{2}$$

b) For polar mounts:

$$\varDelta\tau_{therm.i} = \frac{1}{c} \cdot \Big[\gamma_f \cdot (T(t - \varDelta t_f) - T_0) \cdot (h_f \cdot \sin\varepsilon)$$

$$
\begin{aligned}
&+\gamma_a \cdot (T(t-\Delta t_a) - T_0) \cdot (h_p \cdot \sin\varepsilon \\
&+AO \cdot \cos\delta + h_v - F_a \cdot h_s)\Big]. \qquad (3)
\end{aligned}
$$

c) For polar mounts with axis displaced (Skurikhina 2001):

$$
\begin{aligned}
\Delta\tau_{therm.i} = \frac{1}{c} \cdot \Big[&\gamma_f \cdot (T(t-\Delta t_f) - T_0) \cdot (h_f \cdot \sin\varepsilon) \\
&+\gamma_a \cdot (T(t-\Delta t_a) - T_0) \cdot (h_p \cdot \sin\varepsilon \\
&+AO \cdot \sqrt{1 - [\sin\varepsilon \cdot \sin\phi_0 + \cos\varepsilon \cdot \cos\phi_0 \cdot (\cos\alpha \cdot \cos\Delta\lambda + \sin\alpha \cdot \sin\Delta\lambda)]^2} \\
&+h_v - F_a \cdot h_s)\Big]. \qquad (4)
\end{aligned}
$$

with $\Delta\lambda$ being the error of the fixed axis if it is not oriented due north and ϕ_0 being the inclination of the fixed axis w.r.t. the local horizon. In the case of Richmond these angles are -0.12° and 39.06°, respectively.

d) For X/Y mounts, primary axis north-south:

$$
\begin{aligned}
\Delta\tau_{therm.i} = \frac{1}{c} \cdot \Big[&\gamma_f \cdot (T(t-\Delta t_f) - T_0) \cdot (h_f \cdot \sin\varepsilon) \\
&+\gamma_a \cdot (T(t-\Delta t_a) - T_0) \cdot (h_p \cdot \sin\varepsilon \\
&+AO \cdot \sqrt{1 - (\cos\varepsilon \cdot \cos\alpha)^2} + h_v - F_a \cdot h_s)\Big]. \qquad (5)
\end{aligned}
$$

e) For X/Y mounts, primary axis east-west

$$
\begin{aligned}
\Delta\tau_{therm.i} = \frac{1}{c} \cdot \Big[&\gamma_f \cdot (T(t-\Delta t_f) - T_0) \cdot (h_f \cdot \sin\varepsilon) \\
&+\gamma_a \cdot (T(t-\Delta t_a) - T_0) \cdot (h_p \cdot \sin\varepsilon \\
&+AO \cdot \sqrt{1 - (\cos\varepsilon \cdot \sin\alpha)^2} + h_v - F_a \cdot h_s)\Big]. \qquad (6)
\end{aligned}
$$

It should be noted that $\Delta\tau_{therm.i}$ is the total contribution of the thermal expansion effect at one telescope. With a negative sign it can be used as a correction for the observed time delays.

3 Consequences

The contributions of thermal expansion have to be computed independently for each VLBI delay observation and for each of the two telescopes forming a baseline. With the VLBI time delay being defined as

$$\tau = T_2 - T_1. \qquad (7)$$

with T_1 and T_2 being the generalized arrival times of the signals at site 1 and 2, respectively, the observed delay has to be corrected by subtracting $\Delta\tau_{therm.baseline}$ with

$$\Delta\tau_{therm.baseline} = \Delta\tau_{therm.1} - \Delta\tau_{therm.2}. \qquad (8)$$

The magnitude of annual or daily variations in the topocentric height components depend primarily on the height of the fixed structures (h_f and h_p) and the amplitude of the temperature variations. The 100 m telescope of the Max Planck Institute for Radio Astronomy at Effelsberg (EFLSBERG) with an elevation axis height of 50 m may endure temperature minima of about -5°C and maxima of about 28°C. This alone causes an annual variation in height of almost 20 mm. All other telescopes scale down considerably depending on their dimensions and their temperature variations. Since most antennas used for geodetic and astrometric VLBI belong to a 32 meter class or smaller, their height variations lie in the range of 4 - 6 mm.

When the corrections for thermal expansion are applied consistently to all VLBI observations, no change is expected for most of the mean site coordinates determined by VLBI in the framework of terrestrial reference frame solutions, e.g. for the ITRF, since the reference temperature should represent the long term mean at a particular site. However, if GPT does not exactly equal the real mean temperature, a virtual displacement in the height component may appear. Tab. 1 lists the differences between the GPT model temperatures and the mean temperatures extracted from VLBI log files over a long time span (Malkin, pers. comm.) as well as the respective vertical effect. Sites like Yellowknife, SEST, Maryland Point or Tidbinbilla (64 m) are omitted since they only contributed very few observing sessions in the 1980s and their average temperatures from log file analysis are biased for seasonal effects. Although the remaining differences in temperature of up to 9° C appear large, their effect does not exceed 1.5 mm in height. Moreover, only 3 telescopes exceed the 1 mm threshold. Therefore, it is quite reasonable to start using the mean temperatures from GPT as listed in column 3 of Tab. 1 as reference temperature for thermal expansion corrections. Through this, the VLBI input to future realisations of the ITRF will not be affected in a systematic way other than reducing the annual signatures of the vertical coordinate components.

For completeness, radio telescopes under radomes like Westford, Onsala, Haystack or Syowa should also be mentioned here. They are not affected by the outside air temperature directly but by the temperature behaviour inside the radome that is usually varying less than the outside temperature. Thus, radome enclosed telescopes usually are affected by less thermal deformation than telescopes without radomes. The temperature inside the radome is often difficult to retrieve since mostly only the outside air temperature is monitored for refraction modelling. In addition, the interior of a radome may be heated in winter, at least to a certian extent, removing part of the annual variations. The best way to correct the observations of these antennas is to use in-situ invar wire measurements as has been demonstrated at Onsala (Johansson et al. 1996) or Wettzell (Zernecke 1999). However, if such equipment is not available the temperature inside the radome should be monitored carefully and the expansion effects be modelled accordingly.

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Table 1 Temperature differences between the mean GPT model temperatures and the mean log file temperatures for observing sites with reasonable observing history together with the effect on the vertical position; sorted by temperature difference

Telescope	Temp. from Log files [°C]	Temp. from GPT [°C]	Difference [°C]	Effect in Vertical [mm]
FD-VLBA	17.0	25.9	8.9	1.5
DSS45	13.0	18.5	5.5	1.1
DSS15	16.0	19.0	3.0	0.6
DSS65	15.0	17.9	2.9	0.6
NL-VLBA	11.0	13.9	2.9	0.5
BR-VLBA	12.0	14.0	2.0	0.3
EFLSBERG	10.0	11.8	1.8	1.1
URUMQI	3.0	4.5	1.5	0.2
NOTO	19.0	19.9	0.9	0.2
MEDICINA	14.0	14.6	0.6	0.1
NRAO-140	9.0	9.4	0.4	0.1
NRAO20	9.0	9.4	0.4	0.1
NRAO85-3	9.0	9.4	0.4	0.1
HOBART26	13.0	13.2	0.2	0.0
KAUAI	17.0	17.0	0.0	0.0
MATERA	14.0	14.0	0.0	0.0
PENTICTN	10.0	10.0	0.0	0.0
KOKEE	17.0	16.9	-0.1	0.0
SESHAN25	19.0	18.9	-0.1	0.0
TIGOWTZL	8.0	7.9	-0.1	0.0
WETTZELL	8.0	7.8	-0.2	0.0
RICHMOND	25.0	24.6	-0.4	-0.1
HATCREEK	10.0	9.3	-0.7	-0.1
GOLDVENU	20.0	19.0	-1.0	-0.2
PIETOWN	9.0	7.7	-1.3	-0.2
KASHIM34	16.0	14.5	-1.5	-0.3
KASHIMA	16.0	14.5	-1.5	-0.2
SC-VLBA	27.0	25.5	-1.5	-0.2
OHIGGINS	-1.0	-2.6	-1.6	-0.1
CRIMEA	16.0	14.1	-1.9	-0.3
HARTRAO	18.0	16.1	-1.9	-0.3
LA-VLBA	12.0	10.1	-1.9	-0.3
NYALES20	-2.0	-4.1	-2.1	-0.3
FORTLEZA	29.0	26.7	-2.3	-0.1
ROBLED32	16.0	12.4	-3.6	-0.7
HN-VLBA	11.0	7.1	-3.9	-0.6
OVRO-130	14.0	10.0	-4.0	-1.0
KP-VLBA	15.0	10.2	-4.8	-0.8
MOJAVE12	18.0	13.1	-4.9	-0.4
OV-VLBA	15.0	10.1	-4.9	-0.8
SANTIA12	17.0	11.8	-5.2	-0.4
ALGOPARK	10.0	4.7	-5.3	-0.6
GILCREEK	2.0	-3.3	-5.3	-0.8
MK-VLBA	6.4	0.6	-5.8	-1.0

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