Combined solution C04 for Earth Rotation Parameters consistent with International Terrestrial Reference Frame 2014

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Abstract

The Earth Orientation Center of the IERS, located at Paris Observatory, SYRTE, has the task to provide to the scientific community the international reference time series for the Earth Rotation Parameters (ERP), called "IERS C04" (Combined 04), resulting from a combination of intra-technique ERP series, each of them associated with a given space geodetic technique. The procedure developed to derive the C04 solution was recently upgraded. First, it has been necessary to re-align the solution with respect the axes to the most recent ITRF version, namely ITRF 2014. The past version was aligned on a ITRF released in 2008. Since this epoch there has been a slow degradation of the overall consistency and discrepancies at the level of 30 microarcseconds (μas) have been noticed between the ITRF 2008 C04 and the pole coordinate series associated with ITRF 2014 realization. We have taken this opportunity to upgrade the numerical combination procedure. Improvements mostly concern a more realist weighting of the intra-technique solutions included in the combination, and better handling of nutation offsets. The values of the new C04, denominated 14 C04, have been reprocessed since 1984. The pole coordinates are now fully consistent with ITRF 2014. The nutation offsets and UT1 are made consistent with the International Celestial Reference Frame (ICRF). Moreover the new C04 series better take advantage of intra-technique solutions: over the period 2010-2015 differences 14 C04 - IVS combined series have a standard deviation of 40 μas for nutation (gain of 2 with respect to 08 C04 - IVS dispersion) and 10 μs for UT1, pole coordinates 14 C04 - IGS have a dispersion of about 30 μas (gain of 1.3 with respect to 08 C04 - IGS dispersion). The upgraded C04 solution, updated two times per week, is the official C04 solution since January 2017.

Keywords. Earth rotation, combination, terrestrial reference frame

1 The C04 Combination

To get rid of the inconsistencies pertaining to techniques, multi-technique combination at observation level has been actively promoted by IERS. Among the most active groups, cite DGFI and GRGS. The GRGS has developed the GINS− software, yielding consistent set of normal equations for the 4 geodetic techniques; these normal equations are combined and inverted by software DYNAMO at Paris Observatory for determining consistent astro-geodynamic parameters: the 6 EOP, station coordinates of the observation networks, and possibly radio-sources coordinates.

A less heavy method consists in combining normal equations coming from different centres. It is applied at intra-technique level (case of IVS, IGS, ILRS combined solution). But a multi-technique treatment is more rare. Carried out when updating the International Reference Terrestrial Frame (ITRF) for instance, it allows to derive consistent pole coordinates in the ITRF. As it is not yet operational, the present procedure for producing the reference solution remains a posteriori combination of various intra-techniques EOP series, realigned in the ITRF and International Celestial Reference Frame (ICRF). Final values, namely 30 days from now, are given by the Paris Observatory solution or C04 combined series, starting in 1962; it is obtained from the combination of "operational" EOP series derived from the various astro-geodetic techniques. Production of rapid EOP values is entrusted to USNO with a similar approach.

The combination performed twice a week (Tuesdays and Thursdays) consists in EOP series given at one-day intervals for each of those parameters. Of all EOP time derivatives, the C04 provides only one, namely LOD, widely determined by all satellite techniques. The others can be possibly provided by future combination.

As mentioned earlier, these C04 series have to be consistent ITRF and ICRF. For any rotation of those reference frames produced biases on EOP. In 2008 C04 was recomputed and made consistent with the ITRF 2008 [4], and was denominated 08 C04. Network effects, modelled as discontinuous piece wise function, were extrapolated since this epoch. But the release of ITRF 2014 [2] shows that 08 C04 pole coordinate y, has a bias of about 50 μas with respect to EOP solution estimated consistently with the displacements of the geodetic station defining ITRF 2014, as evidenced by Fig. 2. This reference series for pole coordinates will be denominated ITRF14 in the following.

Such a bias, above the dispersion of GNSS pole coordinate series, is significant. This pushed us to revise the C04 procedure, and provide new C04 values back to 1984, consistent with ITRF 2014.

The objective of this technical note is twofold : on one hand detail the C04 combination procedure and the recent improvements brought in the software code, on the other hand present the new EOP C04 solution, noted 14 C04, its accuracy, and how it is made consistent with ITRF 2014.

2 Description of the C04 procedure

The C04 algorithm can be summed up by the following steps:

Step 0 : Selection of a set of N operational series. This choice is not easy, it has varied over time. For now IERS recommends to take one or two representative series for each technique (operational values completed eventually by a final ones). We do this for pole coordinates and LOD, but for nutation offsets and UT1 we include the whole set of VLBI series validated by the International VLBI Service (IVS).

The selected EOP series entering into the combination have to be made "consistent" as far as possible. This is the object of the steps 1 and 2, detailed now.

Step 1 : Rescaling of the formal uncertainties. The first inconsistency affecting individual EOP concern their formal uncertainty. Most often they do not fit the discrepancy between series. So, prior to combination, they are rescaled as follows. Over a given time interval T, 1 year in the present C04 procedure, selecting series of the same technique, all paired differences with respect to a given reference series are computed as well the corresponding standard deviations (STD_i for series i). The mean, noted STD_m, sums up the intra-technique discrepancy for the considered EOP. On the other hand, EOP of series i is given with its formal error $e_i(t)$ at date t. We compute the corresponding mean e_m^i over the time interval T. If discrepancies between series is purely random, then STD_m is a rough estimate of the mean uncertainty, and it can be compared with the mean formal error e_m^i. This leads to rescale the formal error $e_i(t)$ by STD_m/e_m^i. However, such a method acts to privilege the similar series and discard series, which have singular behaviour with respect to the others. Similarity does not insure that the corresponding EOP values are better, for it can stem from similar processing. In this respect we select series obtained by different software in order to mitigate such a possible bias in rescaling the formal error.

Step 2: EOP series are made consistent with the ICRF and ITRF. Any reference EOP has to be consistent with the International Celestial Reference Frame (ICRF) and the International Terrestrial Reference Frame (ITRF). But the operational series are not perfectly aligned with the ITRF and ICRF. They are often referred to different sub-network of ITRF stations and restricted coverage of the quasars defining the ICRF. Because of these pitfall, the intra-technique EOP series can drift in a "non-controlled" way with respect to ITRF and ICRF (Zhu and Mueller, 1983, IERS Annual Report 2000). In the late eighties, inconsistencies were as large as 1 mas, they are now reduced to less than 100 μas for (x,y), 10 μs for UT1, and 50 μas for (dX,dY) but they are still significant. Before the combination is performed, it is necessary to reset all series on the ICRF and ITRF by assuming

- that the celestial pole offsets (dX, dY) of the IVS combined solution yield the direction of the CIP in the ICRF without any significant decadal trend ;

- that the $UT1-UTC$ of the IVS combined solution yield the Earth rotation angle around of the CIP without any significant drift and bias ;
- that the pole coordinates (x,y) obtained along with the ITRF 2014 solution [2] give the direction of the CIP in the ITRF without decadal trend.

In other words some ”outstanding” series are already consistent with ITRF or ICRF, and are used as guideline for operational series to be combined. Any non-consistent EOP series will present decadal trend with respect to the guide series. The difficulty is to define what we mean by ”decadal trend”. We can assume this is the frequency content above one decade at least. Any filtering method introduces edge effects. For the sake of simplicity, the trend is modelled by a continuous piece-wise linear function according to the time interval reported in Table 1. For each EOP series, entering into the C04 solution, the piece-wise linear function is fitted and removed. Such a modelled drift can be easily extrapolated to the dates of the new data provided by IGS or IVS, and can be maintained over some years.

EOP	dX, dY, UT1	x,y
Reference Series	IVS combined solution	EOP ITRF 2014 (IGN)
Intervals of piece-wise linear fit	1984 – 1993 – 2000 – 2010 – 2015	1984 – 1993 – 2000 – 2010 – 2015

Table 1: Guide series for translating EOP individual series in ITRF and ICRF. Inconsistent reference frames causes long term drift of the individual series with respect to “Guide” series consistent with ICRF and ITRF. These drift are modelled as continuous piece-wise linear function over the consecutive time intervals [1984−1993], [1993−2000], [2000 − 2010], [2010 − 2015] and fitted in the observed differences individual series - guide series.

Step 3: Differences individual series – reference values. As combination of the EOP values involve interpolation, filtering, and other numerical calculation, the larger is the EOP value the larger is the error brought by the numerical procedure. In order to avoid such a pitfall, we combine the differences of the selected EOP series with respect to some guide series, containing the main part of the signal. This guide is nothing else than the former combined solution obtained in a previous run, extended by a prediction. To achieve this, the guide series is smoothed by Vondrak procedure, then linearly intEOPolated for each date of the operational series. Thus, the basic quantities to be combined are the difference between operational series and smoothed-interpolated reference values.

This treatment pertains only to pole coordinates: handling of $UT1$ and nutation offsets deserve some modifications:

- Leap second jumps being inappropriate for numerical treatment, $UT1 - UTC$ values for both individual series and guide series are converted into continuous $UT1 - TAI$ time series before any treatment. Moreover the obtained $UT1-TAI$ values are corrected from the regular variations caused by zonal tides by removing the corresponding IERS model at the date of the sampling. Then differences can be computed.

- In contrast to the pole coordinates and $\Omega(UT1-TAI)$, nutation offsets (dX, dY) are quantities smaller than 1 mas. No need of removing some guide celestial pole offsets: the IAU 2000 precession-nutation-model constitutes the reference itself and the corresponding series are ready for combination. Note that, if celestial pole offsets are provided with respect to IAU 1980 model, they are beforehand transformed into (dX, dY) / IAU 2000.

Step 4: The trend of LOD determined by satellite techniques is made consistent with UT1 obtained by VLBI. The trend of the LOD series derived from GNSS and SLR series cannot be trusted: because of non-modelled instabilities in the satellite orbits, LOD is severely drifting in an unpredictable way with respect to UT1 time derivative. For the same time interval GNSS or SLR series present inconsistent drifts. As the LOD guide of step 3 is already aligned on VLBI-determined UT1, any variations in the corresponding differences with time scale larger than two weeks is considered as spurious. They are isolated by low pass Vondrak filtering (95% of the signal is let at 30 days) [12] [13] and removed from each LOD series.

Step 5: Sorting by increasing dates. For a given EOP, the whole set of paired difference values is gathered, then chronologically sorted.

Step 6: Running average. Data are averaged over successive 0.5 day time intervals. For each of those intervals a mean observation date is determined according to the weight of the observations corresponding to his interval. Then, the observed values are propagated to the averaged date using cubic splines interpolation, and their weighted average is based upon the rescaled formal errors obtained in Step 1. The uncertainty of this averaged value is derived from individual formal errors assuming that these ones are independent.

Step 7: Weighting change. Differences between the averaged series and the individual values (of Step 5), are computed as well as the associated Weighted Root Mean Square (WRMS). If for a given mean date, the offset is 2.57 times more than the WRMS, then the weight of the individual value is divided by 10, and the averaging process of Step 6 is redone.

Step 8: Low pass frequency filtering. Vondrak smoothing (Vondrak 1969, 1977) is applied in order to remove spurious frequency variations possibly introduced by previous numerical procedures. Characteristics of the smoothing, according to the epoch of the solution, are reported in Table 2.

Step 9: Interpolation (Lagrange). The smoothed EOP values are interpolated at 1 day intervals.

1984-1993	Smoothing coefficient	10^3	10^3	10^3	$10^{0.4}$
	5% remaining amplitude	1.2 d	1.2 d	1.2 d	3.3 d
	95% remaining amplitude	3.2 d	3.2 d	3.2 d	8.8 d
1993-2000	Smoothing coefficient	10^4	10^3	10^4	10^1
	5 % remaining amplitude	0.8 d	1.2 d	0.8 d	2.6 d
	95% remaining amplitude	2.2 d	3.2 d	2.2 d	7.0 d
2000-2015	Smoothing coefficient	10^5	10^4	10^5	10^1
	5 % remaining amplitude	0.6 d	0.8 d	0.6 d	2.6 d
	95% remaining amplitude	1.52 d	2.2 d	1.5 d	7.0 d

Table 2: Vondrak smoothing coefficient of the EOP

Step 10: Adding back the intermediate series. The final values are obtained by adding back the par of the signal removed in Step 3: i) the "intermediate" guide series for x, y, $UT1$ and LOD ii) the removed models (zonal tidal effect on UT1-TAI/LOD). Finally the $UT1 - TAI$ values are translated into the discontinuous $UT1 - UTC$ series.

Step 11: Storage in the database, possible extension by a prediction. For operational combination, the next solution needs reference values covering the dates of the future observations. Therefore we extend the present series by a prediction, which is not documented here. The solution associated with its prediction is archived in our data base.

3 Upgrades of the C04 processing associated with the 14 C04 version

The processing for computing the C04 has been upgraded.

First, we get rid of the "network effect" by estimating and removing continuous piece wise linear functions from intra-technique solutions over a period of 31 years (1984-2015) estimated with respect to the guide series of step 2, namely ITRF 2014 and IVS combined series (in contrast, piece wise linear functions considered for 08 C04 version are discontinuous). This leads to an improved consistency and stronger long-term stability of the solution, that will be confirmed by Allan deviation analysis in the next section. As shown in Fig. 2 the visible bias of about 50 μas affecting the y coordinate of the 08 C04 from 2011 does not appear any more in 14 C04 − ITRF 14 differences, with a standard deviation significantly smaller for the new solution.

We set up the direct combination of dX, dY nutation offsets (referred to IAU 2000), producing two times smaller dispersion with respect to IVS nutation series, as shown in Table 4.

Finally, the weighting of intra-techniques EOP and rescaling of their formal uncertainties, according to the "step 1" (see section 2), is applied from 1984;

this permit more realist estimates of the combined EOP uncertainties, which the means are reported in Tab. 3.

In Tab. 3 the differences between 14 C04 solution and the 08 C04 solution are sum up by their standard deviation, estimated over the consecutive interval 1984−1993, 1993−2000, 2000−2010, 2010−2015. Until 2000 the differences between both C04 versions are much larger than the mean uncertainty associated with 14 C04 version.

	x (μas)	y (μas)	UT1 (μs)	dX (μas)	dY (μas)	LOD (μs)
1984 – 1993	613	578	180.3	276	288	117
	274	238	22.0	178	193	31
1993 – 2000	175	188	23.7	130	138	21
	108	96	6.8	51	45	18
2000 – 2010	55	47	48.5	87	99	12
	62	59	8.5	38	37	21
2010 – 2015	41	38	23.9	104	88	7
	48	44	9.1	54	45	14

Table 3: Standard deviation of the differences 14 C04 and 08 C04 (1rst line) and average formal uncertainty of the 14 C04 solution (second line) corresponding to 4 successive periods from 1984 to 2015.

4 Comparison of C04 with intra-technique solutions

The 14 C04 better match the intra-technique combined solutions than the 08 C04 version. This is evidenced by Tab. 4, where we report standard deviations of the differences between the C04 solution and intra-technique combined solutions, namely those of the IVS, IGS and ILRS, considering ancient (08 C04) and new solution (14 C04). In this analysis we have also included the multi-technique combined solution associated with the production of the ITRF 2014.

As the IVS series has a averaged time resolution of about 5 days, a low pass filter is applied to C04 in order to make it spectrally consistent with IVS time series, before to compute their differences and the associated standard deviation.

For pole coordinates we gain about 10 μas with respect for IGS solution from 2001. For UT1 a significant gain of 3 μas is noticeable from 2001 with respect to IVS solution. Nutation offsets evidence a improvement of about $30-60$ μas from 1993.

Assuming that guide series ITRF14 and IVS combined series have the best data for pole coordinates and UT1/nutation offsets respectively, to which extent the 14 C04 reproduce theses series? A way to answer to this question is to perform an Allan Deviation (AD) analysis on the differences (see e.g. [11] for definition of Allan variance or its square root, the Allan deviation, and its applications in the

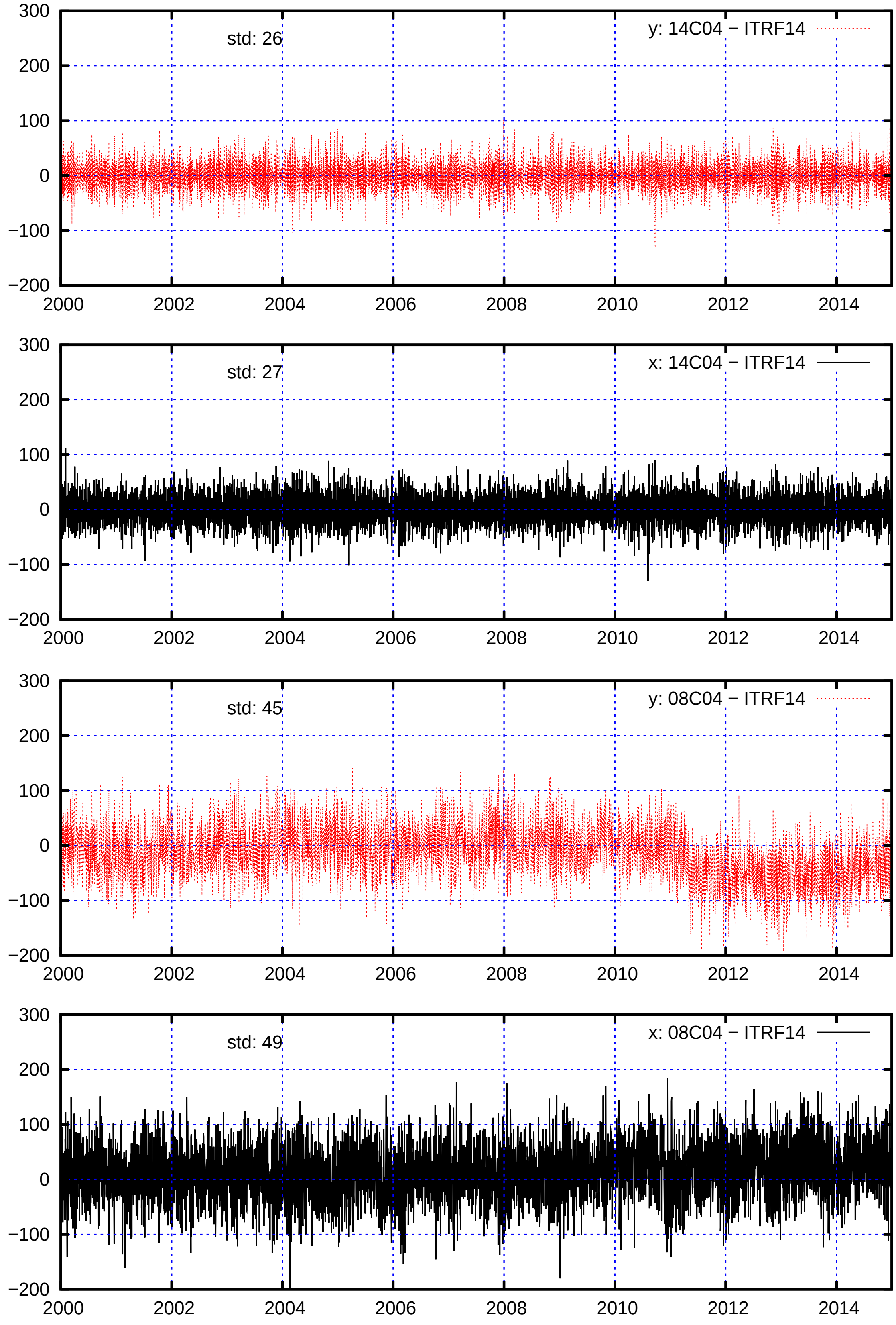

Figure 1: Comparative analysis of 14 C04 and 08 C04 with respect to ITRF14 solution from 2000 to 2015 for pole coordinates x, y.

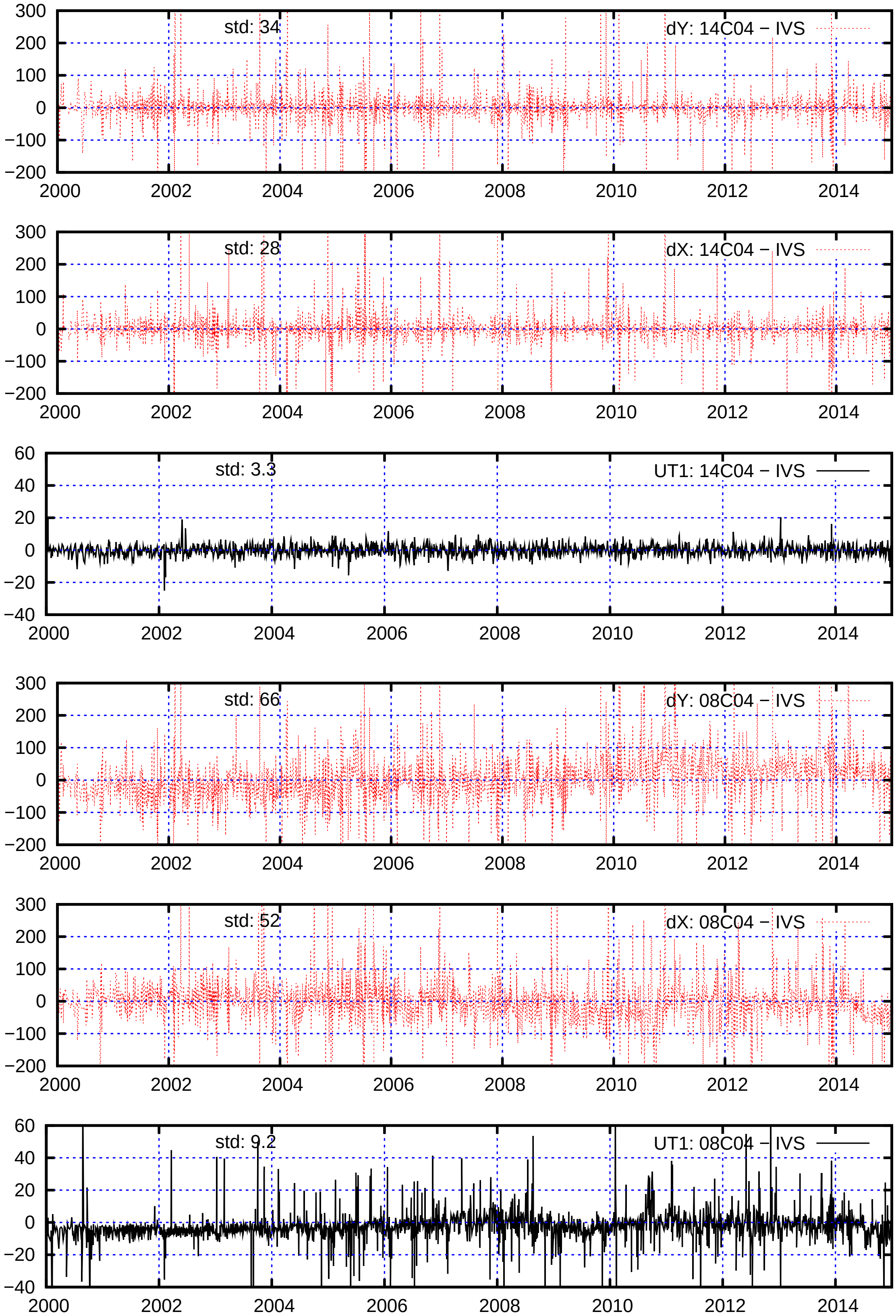

Figure 2: Comparative analysis of 14 C04 and 08 C04 with respect to IVS combined solution from 2000 to 2015 for $UT1$, dX and dY.

C04 ITRF 2014							
		x (μas)	y (μas)	UT1 (μs)	dX (μas)	dY (μas)	LOD (μs)
1984 – 1993	IGS	–	–	–	–	–	–
	IVS	344	291	12.0	100	104	–
	ILRS	–	–	–	–	–	–
	ITRF14	238	202	37.9	–	–	–
1993 – 2001	IGS	–	–	–	–	–	–
	IVS	70	55	4.4	34	37	–
	ILRS	–	–	–	–	–	–
	ITRF14	45	38	10.3	–	–	–
2001 – 2010	IGS	41	33	–	–	–	18
	IVS	68	66	3.3	34	41	–
	ILRS	95	93	–	–	–	22
	ITRF14	75	26	11.9	–	–	–
2010 – 2015	IGS	31	27	–	–	–	10
	IVS	58	56	3.4	21	29	–
	ILRS	77	74	–	–	–	16
	ITRF14	27	25	10.1	–	–	–
08 C04							
		x (μas)	y (μas)	UT1 (μs)	dX (μas)	dY (μas)	LOD (μs)
1984 – 1993	IGS	–	–	–	–	–	–
	IVS	121	110	7.4	90	89	–
	ILRS	–	–	–	–	–	–
	ITRF14	403	372	21.5	–	–	–
1993 – 2001	IGS	–	–	–	–	–	–
	IVS	103	95	9.4	67	60	–
	ILRS	–	–	–	–	–	–
	ITRF14	65	60	10.9	–	–	–
2001 – 2010	IGS	49	37	–	–	–	13
	IVS	73	71	8.3	43	47	–
	ILRS	103	102	–	–	–	18
	ITRF14	47	40	13.2	–	–	–
2010 – 2015	IGS	42	36	–	–	–	11
	IVS	83	74	9.6	56	74	–
	ILRS	107	94	–	–	–	16
	ITRF14	49	49	11.9	–	–	–

Table 4: Standard deviation of the differences of C04 time series with IGS, IVS and ILRS combined series - associated with 4 successive periods from 1984 to 2015. Upper part of the table is for C04 ITRF 2014 (new version). Bottom part is for 08 C04 (ancient version)

field of the astro-geodesy). For LOD reference series is taken as the IGS combined series, because ITRF14 LOD series is too much noisy. The smaller is the Allan deviation at a given time scale, the better C04 reproduce the variations of the corresponding guide series at this time scale. In Fig. 3 we notice that AD of paired difference 14 C04 − guide series is smaller for time scales stretching from a few day to 3000 days, except in the case of LOD where no gain is noticeable. Beyond 1000 days only x coordinate of 08 C04 seems to be slightly better in term of AD. The log-log slope of these AD diagram also indicate the nature of the noise at stake. For dX, dY slope $\sim -1/2$ puts forward a white noise, in agreement with the fact that VLBI series differ one from another by white noise [3]. For $UT1$ white noise behaviour stop after 200 days, and is replaced by Flicker noise (slope 0). Pole coordinates x and y mostly display Flicker noise, characteristic also noticed for the differences between GNSS pole coordinate series in [3]. The fact that AD do not rise beyond 1000 day insure the quality of 14 C04 series at time scale longer from 3 year to 10 days with a noise level of 10 μas.

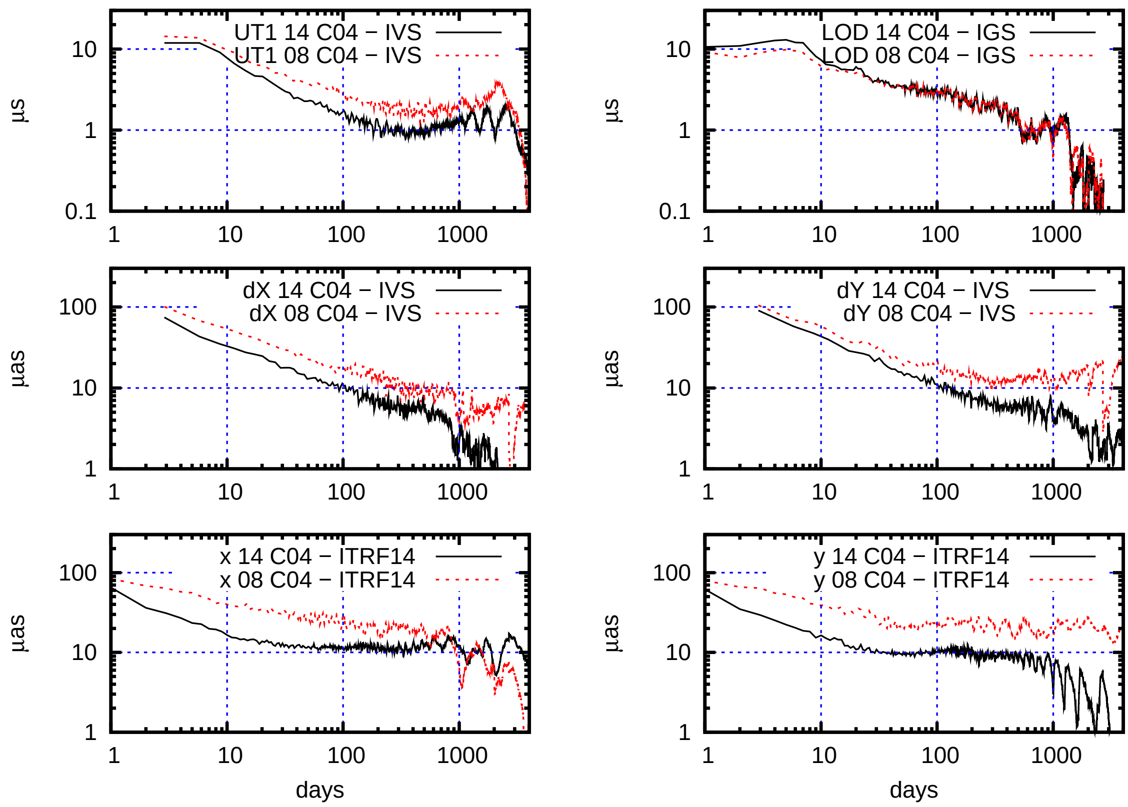

Figure 3: Allan deviation analysis of paired difference series i) C04 − ITRF14 (x,y), ii) C04 − IVS combined solution (dX,dY,UT1), iii) C04 − IGS combined solution (LOD) for both 2008 and 2014 versions.

5 Operational C04 solution

The C04 ITRF 2014 series are recomputed every day taking into account the last EOP determinations over one year back. However, only the last 30 days from now are updated. From 1962 until 30 days from now, C04 EOP are considered as final values. Anyway in case of large errors, we can correct some final values. These final values are monthly published in the IERS Bulletin B, and updated on Paris Observatory server on Tuesday and Thursday. The last 30 days, or rapid EOP, are given in Bulletin A produced by USNO. Of course C04 solution is extended to now, but this extension is not an official IERS product.

In this respect the C04 Combination is achieved by two successive runs, based upon different set of individual series: the first is for final values, the second for rapid values.

In table 5 we check the series entering our present combination according to the distinction "final"/"rapid" EOP. Note that except for SLR, the combination is not restricted to intra-technique combined series. These series eventually mitigate the errors of IGS rapid solution for the last days. For UT1 and nutation offsets, almost the full set of IVS VLBI solutions are considered, for IVS combined series do not consider the whole set of sessions. Rapid UT1 values largely benefit of the intensive UT1 determination. For the rapid pole coordinate and LOD, the set is not restricted to IVS, ILRS and IGS-Rapid solutions. Low weighted CODE are introduced in order to provide eventually lacking data in IGS rapid files, especially for the last days.

6 Summary

The C04 solution has been made consistent with ICRF2 and ITRF 2014 by modelling network effect as continuous piece wise linear function. The corresponding EOP parameters present a better accuracy, and better take advantage of accuracy of VLBI nutation/UT1 data and GNSS pole coordinate/LOD data than the previous C04 version. First nutation/UT1 IVS of 14 C04 is in better agreement with IVS combined solution associated with the second release of the International Celestial Reference Frame (IRCF2) especially at long time scale. Moreover 14 C04 pole coordinates better reproduce the solution with the release of ITRF14 (for Allan deviation gain larger than 10μ as at time scale ranging from 10 to 3000 days). Over the last 15 years the mean uncertainties of the pole coordinates ($\sigma_{x/y} \leq 40$ μas) and LOD ($\sigma_{LOD} \leq 13$ μas) correspond to the standard deviations with respect to the IGS combined series ($std = 30$ μas for pole coordinates, $std = 10$ μs for LOD). A similar conclusion can made on mean uncertainties of $UT1$ ($\sigma_{UT1} = 10$ μas) and dX/dY ($\sigma_{dX/dY} \leq 43$ μas) compared to the standard deviation of C04 with respect to IVS series ($std = 11$ μs for $UT1$ and $std = 25/34$ μas for nutation offsets).

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	x	y	UT1	dX	dY	LOD
VLBI IVS-R	F/R	F/R	F/R	F/R	F/R	
VLBI			F/R	F/R	F/R	
AUS			F/R	F/R	F/R	
BKG			F/R	F/R	F/R	
GSFC			F/R	F/R	F/R	
IAA			F/R	F/R	F/R	
OPA			F/R	F/R	F/R	
USNO			F/R	F/R	F/R	
VLBI Int. BKG I			R			
GSFC I			R			
IAA I			R			
OPA I			R			
USNO I			R			
PUL I			R			
GSI I			R			
GNSS IGS-Rapid	R	R				F/R
IGS-Final	F/R	F/R				F/R
CODE	R	R				R
SLR ILRS	F/R	F/R				F/R

Table 5: EOP series used for the C04 solution from 2015. "F/R" means the corresponding EOP series is selected for both obtaining final (F) and rapid (R) solution, "R" that this series is only for getting rapid values.

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