6 Geopotential (Draft new section based on R. Biancale initial text (Aug 2008) and S. Bettadpur proposal (Feb 2009): Trial by GP 22 April 2010, modified SB 28 April, GP 21 May, BES 7 June, SB 14 June)

6.3 Effect of the Ocean Tides

The dynamical effects of ocean tides are most easily incorporated as periodic variations in the normalized Stokes' coefficients of degree n and order m $\Delta\bar{C}_{nm}$ and $\Delta\bar{S}_{nm}$. These variations can be evaluated as

$$[\Delta\bar{C}_{nm} - i\Delta\bar{S}_{nm}](t) = \sum_f \sum_{+}^{-} (\mathcal{C}^{\pm}_{f,nm} \mp i\mathcal{S}^{\pm}_{f,nm}) e^{\pm i\theta_f(t)}, \tag{14}$$

where $\mathcal{C}^{\pm}_{f,nm}$ and $\mathcal{S}^{\pm}_{f,nm}$ are the geopotential harmonic amplitudes (see more information below) for the tide constituent f, and where $\theta_f(t)$ is the argument of the tide constituent f as defined in the explanatory text below Equation (7).

Ocean tide models are typically developed and distributed as gridded maps of tide height amplitudes. These models provide in-phase and quadrature amplitudes of tide heights for selected, main tidal frequencies (or main tidal waves), on a variable grid spacing over the oceans. Using standard methods of spherical harmonic decomposition and with the use of an Earth loading model, the maps of ocean tide height amplitudes have been converted to spherical harmonic coefficients of the geopotential, and provided for direct use in equation (14). This computation follows Equation (20) and has been carried out for the tide model[s] proposed in section 6.3.2.

Typically, an ocean tide model provides maps for only the largest tides or main waves. The spectrum of tidal geopotential perturbations can be completed by interpolation from the main waves to the smaller, secondary waves, using an assumption of linear variation of tidal admittance between closely spaced tidal frequencies. For each secondary wave, the geopotential harmonic amplitudes can be derived from the amplitudes of two nearby main lines, or pivot waves, (labeled with subscripts 1 and 2) as [I ASSUME IT MUST BE SOMETHING LIKE - This must be verified by RB]:

$$\begin{aligned} \mathcal{C}^{\pm}_{f,nm} &= \frac{\dot\theta_f - \dot\theta_1}{\dot\theta_2 - \dot\theta_1} . \frac{H_f}{H_2} \mathcal{C}^{\pm}_{2,nm} + \frac{\dot\theta_2 - \dot\theta_f}{\dot\theta_2 - \dot\theta_1} . \frac{H_f}{H_1} \mathcal{C}^{\pm}_{1,nm} \\ \mathcal{S}^{\pm}_{f,nm} &= \frac{\dot\theta_f - \dot\theta_1}{\dot\theta_2 - \dot\theta_1} . \frac{H_f}{H_2} \mathcal{S}^{\pm}_{2,nm} + \frac{\dot\theta_2 - \dot\theta_f}{\dot\theta_2 - \dot\theta_1} . \frac{H_f}{H_1} \mathcal{S}^{\pm}_{1,nm} \end{aligned} \tag{15}$$

where H is the astronomic amplitude of the considered wave. See an example in Table 6.7 developped for the main waves of FES2004 (see section 6.3.2).

Some background information on the determination of the coefficients is given in section 6.3.1, and is included here for completeness. It is not necessary for the evaluation of tidal perturbations to the geopotential. Information on selected tidal models and their use is provided in section 6.3.2.

6.3.1 Background on ocean tides models

Ocean tides models are conventionally expressed in terms of amplitude and phase of waves at certain discrete frequencies.

$$\xi(\phi, \lambda, t) = \sum_f Z_f(\phi, \lambda) \cos(\theta_f(t) - \psi_f(\phi, \lambda)) \tag{16}$$

where Z_f is the amplitude of wave f, ψ_f is the phase at Greenwich and θ_f is the Doodson's argument, see the explanatory text below Equation (7).

When expanding amplitudes (Z_f) and phases (ψ_f) of the different waves of tides (from cotidal grids) in spherical harmonic functions of $Z_f cos(\psi_f)$ and $Z_f sin(\psi_f)$, it yields:

$$\xi(\phi,\lambda,t) = \sum_f \sum_{n=1}^{N} \sum_{m=0}^{n} \bar{P}_{nm}(\sin\phi) \sum_{+}^{-} \xi^{\pm}_{f,nm}(\lambda,t) \tag{17}$$

where

$$\xi^{\pm}_{f,nm}(\lambda,t) = \bar{C}^{\pm}_{f,nm}\cos(\theta_f + \chi_f \pm m\lambda) + \bar{S}^{\pm}_{f,nm}\sin(\theta_f + \chi_f \pm m\lambda) \tag{18}$$

The couples of coefficients $\left(\bar{C}^{\pm}_{f,nm}, \bar{S}^{\pm}_{f,nm}\right)$ represent prograde and retrograde normalized spherical harmonic coefficients of the main wave f at degree n and order m, and can be alternately expressed in terms of amplitude $\hat{\bar{C}}^{\pm}_{f,nm}$ and phase $\varepsilon^{\pm}_{f,nm}$) such as:

$$\begin{aligned} \bar{C}^{\pm}_{f,nm} &= \hat{\bar{C}}^{\pm}_{f,nm}\sin(\varepsilon^{\pm}_{f,nm}) \\ \bar{S}^{\pm}_{f,nm} &= \hat{\bar{C}}^{\pm}_{f,nm}\cos(\varepsilon^{\pm}_{f,nm}) \end{aligned} \tag{19}$$

The χ_f values agree with the so-called Shureman convention which is traditionally applied in cotidal maps. They comply with the Doodson-Warburg convention which is defined according to the sign of the harmonic amplitude H_f (see Table 6.6 according to Cartwright and Eden, 1973).

Table 6.6: Values of the phase bias χ_f according to the sign of H_f

	$H_f > 0$	$H_f < 0$
n_1=0, long period wave	π	0
n_1=1, diurnal wave	$\frac{\pi}{2}$	$-\frac{\pi}{2}$
n_1=2, semi-diurnal wave	0	π

For each wave f, the coeeficients $\mathcal{C}^{\pm}_{f,nm}$ and $\mathcal{S}^{\pm}_{f,nm}$ to be used in equation (14) can be computed as

$$\begin{aligned} \mathcal{C}^{\pm}_{f,nm} &= \tfrac{4\pi G\rho_w}{g_e}(\tfrac{1+k'_n}{2n+1})\hat{\bar{C}}^{\pm}_{f,nm}\sin(\varepsilon^{\pm}_{f,nm} + \chi_f) \\ \mathcal{S}^{\pm}_{f,nm} &= \tfrac{4\pi G\rho_w}{g_e}(\tfrac{1+k'_n}{2n+1})\hat{\bar{C}}^{\pm}_{f,nm}\cos(\varepsilon^{\pm}_{f,nm} + \chi_f) \end{aligned} \tag{20}$$

where G and g_e are given in Chapter 1, ρ_w is the density of seawater (1025 kg m^{-3}) and where k'_n is the load deformation coefficient of degree n (k'_2=−0.3075, k'_3=−0.195, k'_4=−0.132, k'_5=−0.1032, k'_6=−0.0892)).

6.3.2 Ocean tides model(s)

The practical implementation of ocean tide models in this form begins with identification of the underlying ocean tide height model. Once this model is identified, further needed information can include the specification of maximum degree and order of the expansion; the identification of the pivot waves for interpolation; the special handling (if necessary) of the solar (radiational) tides, or the long-period tidal bands.

For the case of the FES2004 ocean tide model, these details of implementation are provided next. [TO BE EXPANDED IF DIFFERENT CONVENTIONAL USAGES]

FES2004

The FES2004 ocean tides model (F. Lyard et al., 2006) includes long period waves (Sa, Ssa, Mm, Mf, Mtm, Msqm), diurnal waves (Q1, O1, P1, K1), semi-diurnal waves (2N2, N2, M2, S2, K2) and the quarter-diurnal wave (M4 [1]). For direct use in equation (14), the coefficients $\mathcal{C}^{\pm}_{f,nm}$ and $\mathcal{S}^{\pm}_{f,nm}$ for the main tidal waves of FES2004 can be be found at <[2]> [THIS IS A TEST COMPUTATION AND THESE RESULTS SHOULD NOT BE FULLY TRUSTED AT THIS TIME].

The tide height coefficients can be found in the file <[3]> , both in the form of the coefficients $\bar{C}^{\pm}_{f,nm}$ and $\bar{S}^{\pm}_{f,nm}$ and in the form of the amplitudes $\hat{\bar{C}}^{\pm}_{f,nm}$ and phases $\varepsilon^{\pm}_{f,nm}$, as defined in Equations (18) and (19). They have been computed up to degree and order 100 by quadrature method from quarter-degree cotidal grids. Then ellipsoidal corrections to spherical harmonics were applied (Balmino, 2003) in order to take into account that tidal models are described on the oblate shape of the Earth.

Table 6.7 provides a list of admittance waves which can be taken into account to complement the model. It indicates the pivot waves for linear interpolation following equation (15), where indexes 1 and 2 refer to the two pivot waves.

It is to be noticed that radiational waves like S1 and S2 require special handling, since the common altimeatric models (including FES2004) for these tides include the contributions of atmospheric pressure variations on the ocean height (i.e. the radiational tide). As a result, neither S1 and S2 are used as pivot waves for interpolation in Table 6.7. While S2 wave is available as a part of the FES2004 model, a mean S1 wave is given outside FES2004 and available in file <[4]>).

The additionally provided mean S1 wave should only be used in case the gravitational influences of mass transport from an ocean circulation models like MOG2D (Carrère and Lyard, 2003) are not also modeled. This is because the S1 signal is generally part of such ocean circulation models provided with an interval of 6 hours.

Moreover very long period waves like $\Omega 1$ *(18.6 yr)* and $\Omega 2$ *(9.3 yr)* which are not yet correctly observed can be modelled as equilibrium waves. Their amplitudes (and phases) are computed from the astronomical amplitude H_f considering the elastic response of the Earth through the Love numbers:

$$\hat{\bar{C}}_{f,20} = \frac{1 + k_2 - h_2}{\sqrt{4\pi}} |H_f|, \quad \epsilon_{f,20} = -\frac{\pi}{2}$$

where $k_2 = 0.29525$ and $h_2 = 0.6078$ are the Love numbers of potential and deformation respectively.

Influence of tidal models

For a satellite like Stella (Altitude 800 km, inclination 98.7 deg. and eccentricity 0.001), for one day of integration, the effects of ocean tides are typically of order several cm and can reach 20 cm. It is estimated that the main waves of the FES2004 model typically represents 80% of the effect (Biancale, 2008).

For Starlette (Characteristics) and Lageos (Characteristics), XXX integration time showed a 3-D RMS difference (mostly along-track) of 9 and 7 mm, respectively, for the difference between FES2004 and the older CSR3.0 ocean tide model (Ries, 2010).

[1] The χ value for M4, not given in Table 6.6, is 0
[2] presently: ftp://tai.bipm.org/iers/temp/FES2004_tides_Biancale/fes2004_Cnm-Snm.dat
[3] presently: ftp://tai.bipm.org/iers/temp/FES2004_tides_Biancale/fes2004.dat
[4] presently: ftp://tai.bipm.org/iers/temp/FES2004_tides_Biancale/S1.dat

Table 6.7: List of astronomical amplitudes H_f (m) for main waves of FES2004 (in bold) and for some secondary waves (with their pivot waves when they have to be linearly interpolated).

Darwin's symbol	Doodson's number	H_f	Pivot wave 1	Pivot wave 2
$\Omega 1$	055.565	.02793		
$\Omega 2$	055.575	-.00027		
S_a	**056.554**	**-.00492**		
S_{sa}	**057.555**	**-.03100**		
S_{ta}	058.554	-.00181	057.555	065.455
M_{sm}	063.655	-.00673	057.555	065.455
	065.445	.00231	057.555	065.455
M_m	**065.455**	**-.03518**		
	065.465	.00229	065.455	075.555
	065.555	-.00375	065.455	075.555
	065.655	.00188	065.455	075.555
M_{sf}	073.555	-.00583	065.455	075.555
	075.355	-.00288	065.455	075.555
M_f	**075.555**	**-.06663**		
	075.565	-.02762	075.555	085.455
	075.575	-.00258	075.555	085.455
M_{stm}	083.655	-.00242	075.555	085.455
	083.665	-.00100	075.555	085.455
M_{tm}	**085.455**	**-.01276**		
	085.465	-.00529	085.455	093.555
M_{sqm}	**093.555**	**-.00204**		
	095.355	-.00169	085.455	093.555
	117.655	-.00194	135.455	145.555
2Q1	125.755	-.00664	135.655	145.555
$\sigma 1$	127.555	-.00802	135.655	145.555
$\sigma 1$	135.645	-.00947	135.655	145.555
Q1	**135.655**	**-.05020**		
	137.445	-.00180	135.655	145.555
$\rho 1$	137.455	-.00954	135.655	145.555
	145.545	-.04946	135.655	145.555
O1	**145.555**	**-.26221**		
	145.755	.00170	145.555	165.555
$\tau 1$	147.555	.00343	145.555	165.555
	153.655	.00194	145.555	165.555
	155.455	.00741	145.555	165.555
	155.555	-.00399	145.555	165.555
M1	155.655	.02062	145.555	165.555
	155.665	.00414	145.555	165.555
$\chi 1$	157.455	.00394	145.555	165.555
$\pi 1$	162.556	-.00714	145.555	165.555
P1	**163.555**	**-.12203**		
S1	164.556	.00289		
K1-	165.545	-.00730	145.555	165.555
K1	**165.555**	**.36878**		
K1+	165.565	.05001	145.555	165.555
$\psi 1$	166.554	.00293	145.555	165.555
$\varphi 1$	167.555	.00525	145.555	165.555
$\theta 1$	173.655	.00395	145.555	165.555
J1	175.455	.02062	145.555	165.555

continued

Darwin's symbol	Doodson's number	H_f	Pivot wave 1	Pivot wave 2
	175.465	.00409	145.555	165.555
So1	183.555	.00342	145.555	165.555
	185.355	.00169	145.555	165.555
Oo1	185.555	.01129	145.555	165.555
	185.565	.00723	145.555	165.555
$\nu1$	195.455	.00216	145.555	165.555
3N2	225.855	.00180	235.755	245.655
$\epsilon2$	227.655	.00467	235.755	245.655
2N2	**235.755**	**.01601**		
$\mu2$	237.555	.01932	235.755	245.655
	245.555	-.00389	237.755	245.655
	245.645	-.00451	237.755	245.655
N2	**245.655**	**.12099**		
$\nu2$	247.455	.02298	245.655	255.555
$\gamma2$	253.755	-.00190	245.655	255.555
$\alpha2$	254.556	-.00218	245.655	255.555
	255.545	-.02358	245.655	255.555
M2	**255.555**	**.63192**		
$\beta2$	256.554	.00192	255.555	275.555
$\lambda2$	263.655	-.00466	255.555	275.555
L2	265.455	-.01786	255.555	275.555
	265.555	.00359	255.555	275.555
	265.655	.00447	255.555	275.555
	265.665	.00197	255.555	275.555
T2	**272.556**	**.01720**	255.555	275.555
S2	**273.555**	**.29400**		
R2	274.554	-.00246	255.555	275.555
K2	**275.555**	**.07996**		
K2+	275.565	.02383	255.555	275.555
K2++	275.575	.00259	255.555	275.555
$\eta2$	285.455	.00447	255.555	275.555
	285.465	.00195	255.555	275.555
M4	**455.555**			

References

Balmino, G., 2003, "Ellipsoidal corrections to spherical harmonics of surface phenomena gravitational effects,," *Special publication in honour of H. Moritz*, Technical University of Graz.

Biancale, R., 2008, personal communication.

Carrère, L. and Lyard, F., 2003, "Modelling the barotropic response of the global ocean to atmospheric wind and pressure forcing comparison with observations," *Geophys. Res. Lett.* **30**(6), p. 1275, DOI 10.1029/2002GL016473.

Cartwright, D. E. and Tayler, R. J., 1971, "New Computations of the Tide-Generating Potential," *Geophys. J. Roy. astr. Soc.*, **23**, pp. 45–74.

Cartwright, D. E. and Eden, A., 1973, " Corrected tables of tidal harmonics," *Geophys. J. Roy. astr. Soc.*, **33**, pp. 253–264.

Casotto, S., 1989, "Ocean Tide Models for TOPEX Precise Orbit Determination," Ph.D. Dissertation, The Univ. of Texas at Austin.

Lyard, F., Lefevre F., Letellier T., Francis O., 2006, "Modelling the global ocean tides: modern insights from FES2004," Ocean Dynamic 56, pp. 394–415.

Ries, J.C., 2010, personal communication.